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LONGEVITY AND GENETIC IMPROVEMENT ISSUES IN ECONOMIC REPLACEMENT STRATEGIES FOR DAIRY COWS UNDER ALBERTA CONDITIONS

by

WAYNE LOHR

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled LONGEVITY AND GENETIC IMPROVEMENT ISSUES IN ECONOMIC REPLACEMENT STRATEGIES FOR DAIRY COWS UNDER ALBERTA CONDITIONS submitted by Wayne Lohr in partial fulfilment of the requirements for the degree of MASTER OF SCIENCE in Agricultural Economics.

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Supervisor

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Date Dec. 7 19 83

Dedication

To my loving wife, Carolyn, and my daughters Ami and Krystal. Without your patience and understanding, this journey would never have been possible nor successful.

The greatest achievement of the human spirit is to live up to one's opportunities and make the most of one's resources.
-Vauvenargues

Abstract

The objective of the study was to identify economic cow replacement strategies for Alberta dairy farms and to subsequently determine if differences exist between the indicated economic replacement strategies and current behavior.

A lifetime milk yield function was determined through an ordinary least squares regression using production data from four herds containing in total 833 cows. A capital asset replacement model which determines the net present value of the net annual residual cashflows from the dairy cow (including changes in salvage value between lactations and a value for the calf produced) was developed. The resultant net present values for each possible lactation cycle were evaluated using a finite Markov process to allow for involuntary cow disposals. This probability weighted net present value was used to identify the economic replacement age as well as the steady state herd age composition.

The study identified the economic replacement time to occur at the end of the third lactation at which time the cow was replaced with a new genetically superior cow. This replacement strategy results in an average herd age of 4.7 years which compares favorably to the actual average age in the Alberta dairy herd (obtained from Alberta Dairy Herd Improvement Services) of 4.78 years of age.

It was concluded that, on average, Alberta dairy farmers are behaving in a rational economic manner as defined by the model.

Further research is needed to better estimate the lifetime lactation curves. The establishment of veterinary costs related to age and lactation stage and how these costs change over a cows productive life also require further study.

Acknowledgements

A NEW CHANCE

We all have to learn, in one way or another that neither men nor boys get second chances in this world. We all get new chances till the end of our lives, but not second chances in the same set of circumstances; and the great difference between one person and another is, how he takes hold of and uses his first chance, and how he takes his fall if it is scored against him.
- Thomas Hughes

To Dr. Mel Lerohl, for his understanding and "fatherly advice" and for providing the opportunity to study in the Department.

A special thank-you to Agriculture Canada and the Canadian Rural Transition Program and specifically to Solange Williams. Only she will realize the important role that she has played in the acquisition of this degree. Her confidence and faith have provided great moral support when it was needed the most. I can only hope that the successful completion of this degree will, in some small way, serve as personal repayment to you and your Department and show that the Program can have positive benefits.

To my parents, for instilling the importance of education and the quest of knowledge; to reach beyond the realms of the obvious for the foundations of a deeper understanding; and above all else, to reach for your personal best and to measure success against your own expectations and aspirations - not those of others. While we may not have always agreed, it is to these humble and simple principles that one always returns for guidance and assurance.

Special thanks to Barb and Hildegard for always knowing where to find the obscure articles and references. As well, a very special thanks to Holly for her words of "wisdom" and for always brightening the days with her presence.

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The past two years have brought new challenges and new friends, sometimes in combination. Glenn Armstrong's insights into the mathematics of the replacement problem using Wooden Cows will be remembered and valued for many years to come. The Maritime view of the world was graciously provided by Diane Dunlop whose friendship and assistance have been greatly appreciated.

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Yes, Krystal. Daddy is finally finished his damned thesis!

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I. Introduction and Background

A. Background

The dairy industry plays a significant role in the agricultural economy of Alberta. In 1985, milk and cream sales amounted to over 246 million dollars or 6.2% of total farm sales. In 1986, milk and cream sales amounted to over 248 million dollars or 6.1% of total farm sales.[1]

Dairy extension specialists have expressed the concern that the rate of turnover of dairy cows in the Alberta dairy herd may be too rapid. The dairy extension specialists have hypothesized that there may be economic benefits available to dairy producers that could be captured by individual dairy producers improving their replacement strategies. This concern may be accentuated when the issue of genetic improvement is introduced into the decision-making framework. The question of the optimum economic life of a dairy cow has also been identified by the Alberta Dairy Seminar Planning Committee as an important issue to the dairy industry of Alberta.

Statistics obtained from the Alberta Dairy Herd Improvement Services indicate that the average age of a dairy cow in Alberta is 4.78 years, with the average age of first calving at 28 months. The average calving interval is 13.5 months. It is suggested by these same statistics that the average time of disposal of dairy cows in Alberta is 5.3 years of age; sometime during the cow's third lactation.

The production cycle of the dairy cow is such that the output of milk from a particular cow increases at a decreasing rate from the time of first calving to maturity, then remains relatively constant over some time period, and subsequently decreases at an increasing rate with senescence. (Smith 1968; Lush and Shrode 1950; and Giaver 1966)

Dairy science literature suggests that a dairy cow reaches maximum production at around seven years of age (Mao, Burnside, Wilton and Freeman 1974; Lush and Shrode 1950) which places the point of attaining maximum production sometime during the fourth or fifth lactation. The production cycle, limited barn space and physical resources of dairymen, quota restrictions and other

---

1 Alberta Agriculture Statistics Handbook, 1985 and 1986.

production constraints create an environment in which the net revenue generated by a mature or aging cow may no longer be sufficient to offset the net revenue that would be generated by a replacement cow, particularly if the replacement cow is genetically superior.

Several studies have been conducted on optimal replacement policies for dairy cows (Gardner, 1980; Giaver, 1966; Jenkins and Halter, 1964; Redman and Kuo, 1969; Smith, 1968; Stewart, 1975; and Van Arendonk, 1984). With the exception of Stewart and Van Arendonk, these studies have not included genetic improvement in the analysis. There appears to be a lack of research dealing with dairy cow economic replacement problems which take genetic improvement and questions of longevity of production into account, particularly as these concerns relate to Alberta producers and conditions (i.e., quota restrictions, climatic conditions, etc.).

B. Purpose of Study

The purpose of this research is to construct an economic asset replacement model for dairy cows that predicts replacement behavior under Alberta conditions. Predictions of this model will be compared to behavior of Alberta dairy producers as revealed by statistics obtained from Alberta Dairy Herd Improvement Services.

C. Outline of Thesis

The first chapter of the study has introduced the problem and discussed the background from which the problem was identified. Chapter 2 reviews relevant literature and economic theory. The analytical model is developed in Chapter 3. The empirical model employed in this research project is outlined in Chapter 4. Results and analysis are discussed in Chapter 5. Chapter 6 concludes the Thesis with the conclusions and a brief summary.

II. Capital Asset Replacement Theory Applied To Dairy Cows

A. Capital Asset Replacement Theory

Capital asset replacement theory has been well developed by a number of disciplines including agriculture, business, forestry, engineering, and economics. Some cases involve an appreciating asset such as a stand of timber while in others a depreciating asset such as machinery is under consideration. Replacement can be with an identical asset or a superior asset. The rules of replacement can change depending on the type of replacement asset considered. Never the less, the goal of the asset manager is assumed to be to maximize the value of net returns over time. Faris (1960), Burt (1965), and Perrin (1972) were instrumental forces in the development of the marginal net revenue approach to optimal replacement policies.

The technique for determining the optimum time for replacement of an asset depends on the shape of the marginal net revenue curve, the characteristics of the subsequent assets, the discount rate and whether or not involuntary replacement takes place (Faris, 1960; Burt, 1965; and Perrin, 1972).

Faris, an early author on optimal replacement strategies, noted that "in a continuous operation, in which each lot (asset) is replaced immediately by a new lot, the operator is concerned with maximizing his *average net revenue over time*" (Faris, 1960).

In the situation where no discounting is used and no involuntary replacement occurs, (Faris, 1960) the optimal replacement time occurs at the point in time where the marginal net revenue from the existing asset is equal to the maximum average net revenue expected from the following identical replacement asset. Thus, when replacement is with an identical asset, the optimal replacement time is when the average (annualized) net revenue is at a maximum. This maximum average net revenue of the replacement asset is the opportunity cost of postponing replacement (see figure 2.1 - based on Faris, 1960).

Once past the point of tangency (maximum average net revenue) with the total net revenue curve, the additions to net revenue (marginal net revenue) are decreasing which begins to pull the average net revenue down from its maximum.

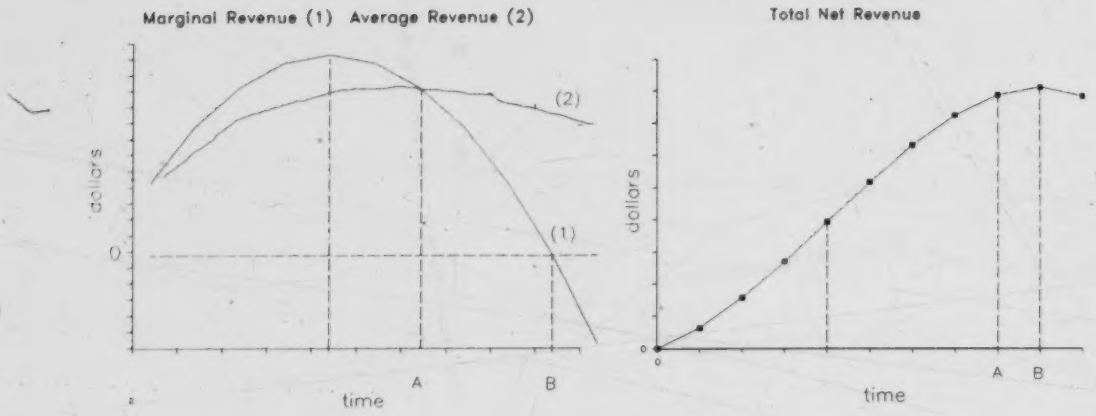

Figure 2.1. Determination of the optimum time for replacement (A) in case of identical replacement.[2] [3]

In the case where changes are anticipated in the net returns function; the rule would be to carry the present lot only to the point where marginal net revenue from the present lot equals maximum average net revenue anticipated from the subsequent lot. To carry the present lot beyond this point would yield additions to net revenue less than the maximum average anticipated in the future. Hence, it would pull the average down.

To make a valid comparison between two assets it is necessary to compare anticipated revenues at the same point in time (usually the present time). Therefore, in order to make the decision of whether or not to replace an asset the operator must first determine the present value of the anticipated net revenue from the replacement asset.

The concept of maximizing the stream or flow of net revenue over time implies that the present value of the net revenue in year "n" can be converted into an equivalent stream of income for the following asset in order to compare it with the marginal net revenue from the present asset. This is accomplished by multiplying the net revenue by the sinking fund formula. [4]

Thus the amortized present value of anticipated net revenue (ANR), can be expressed as:

---

2 The optimum time for replacement (A) is reached earlier than the time at which total net revenue from the present asset is maximized (B).

3 adapted from Van Arendonk (1984). page 331.

4 $FV = A\left[\frac{(1+i)^n - 1}{i}\right]$ ...

$$ANR_n = \left[\frac{r(1+r)^n}{(1+r)^n - 1}\right]\left[\frac{V_1}{(1+r)^1} + \frac{V_2}{(1+r)^2} + \cdots + \frac{V_n}{(1+i)^n}\right]$$

where "r" denotes the discount rate for time preference and "i" denotes the interest rate for annual and initial costs (Faris, 1960).

When the marginal net revenue from the present asset becomes smaller than the anticipated stream of net revenue from the asset immediately following, it is profitable to replace the present asset.

The principle of optimum replacement for enterprises with long production periods and with returns being realized by the sale of the asset can be stated as follows:

> *"The optimum time to replace is when the marginal net revenue from the present asset is equal to the highest amortized present value of anticipated net revenues from the asset immediately following."*

The principle of optimum replacement for enterprises having a long production period and with revenues being realized throughout the life of the asset is the same as that for an enterprise where net revenue is received in a lump sum. However, there is a difference in the method of handling acquisition costs and annual costs.

Because of the flow of revenue from the asset, the cost of acquisition can be repaid in most instances before the asset is replaced. *Therefore*, interest should be compounded only on the unpaid balance of the acquisition cost.

In order to obtain the amortized present value, the net revenue must be accumulated. Thus the amortized present value in year "n" can be expressed as:

$$ANR = \left[\sum_{k=0}^{n} \frac{V_k}{(1+r)^k}\right]\left[\frac{r(1+r)^n}{(1+r)^n - 1}\right]$$

The annual net revenue is also the marginal net revenue. The optimum replacement age, then, is when the annual net revenue from the present asset is equal to the anticipated amortized present value of the net revenue of the following asset (Faris, 1960).

> *The optimum time to replace is when the marginal net revenue from the present enterprise is equal to the highest amortized present value of anticipated net revenue from the following enterprise.*

For enterprises that furnish a flow of net revenues as well as for enterprises where the net revenue is obtained by the sale of the asset, it is necessary to charge interest on the unpaid balance of the acquisition cost if an appropriate net revenue is to be obtained.

Faris (1960) stated that time preference is primarily a function of opportunity costs where the marginal rate of time preference is the slope of an indifference curve relating present to future consumption.

Chisholm (1966) stated that

"The optimum time to replace is when the marginal net revenue from the present enterprise is equal to the highest amortized present value of anticipated net revenues from the enterprise immediately following."

The aim should be to maximize profit per unit of time, which requires equating the marginal net rate of profit per unit of time to the average net rate of profit per unit of time.

It should be noted that the net present value criterion provides an accurate solution if the aim is to select the production period which maximizes the net present value for a perpetual sequence of production periods.

Assuming a time horizon of "t" production periods, this criterion may be expressed as:

$$PNV^* = \max_{n=1}^{t} \left\{ \frac{P_1}{(1+i)^n} + \frac{P_2}{(1+i)^{2n}} + \ldots + \frac{P_t}{(1+i)^{tn}} \right\}$$

where: PNV*: the net PV of net revenue from a perpetual sequence of production periods.

P(t): the net revenue of each period.

i: market rate of interest for outside borrowing and investment.

For a perpetual sequence of production periods, this may be expressed as:

$$PNV^* = \max_{n=1}^{N} \left\{ PNV + \left[\frac{PNV}{(1+i)^n - 1} \right] \right\}$$

where PNV: the PV of net revenue from a single production period.

Where $P_1, P_2, \ldots P_t$ are equal, the preceding criterion can be redefined as follows:

$$A^* = \max_{n=1}^{N} \left\{ P \left[\frac{i}{((1+i)^n - 1)} \right] \right\}$$

where A*: the maximum amortized present value of net revenue.

This is equivalent to the time preference proper criterion given by Winder and Trant (1961). Chisholm presented the argument that this equation embodied not time preference proper, as defined by Winder and Trant, but rather all the relevant opportunity costs of the resources tied up in the production process (Chisholm, 1966).

Perrin (1972) suggested that an asset (the dairy cow is a special class of capital asset) should be kept

"until the gain from keeping the current asset for another time interval no longer exceed the opportunity gains which could be realized from the replacement asset during the same period."

Perrin goes on to elaborate that the simplest assumption to make about the motivation of an asset manager is that he (the asset manager) wishes to maximize the present value of the entire future stream of residual earnings from the productive process associated with the asset and that the replacement problem is to choose a replacement age which maximizes this present value. Future revenues must be discounted to a common point in time if comparing anticipated revenues. (Faris, 1960; Perrin, 1972)

Perrin (1972) contended that an infinite planning horizon had to be used when the marginal net revenue approach was utilized because the approach maximized net revenues of the existing asset and *all* replacement assets.

Perrin (1972) developed the basis upon which to analyze a replacement decision where the replacement assets are different from the present asset. For this situation, the optimum time for replacement of the existing cow will not of necessity coincide with the time at which its average net revenue is at a maximum. For the calculation of opportunity costs associated with postponed replacement, all following replacement assets are assumed identical. (See Figure 2.2)

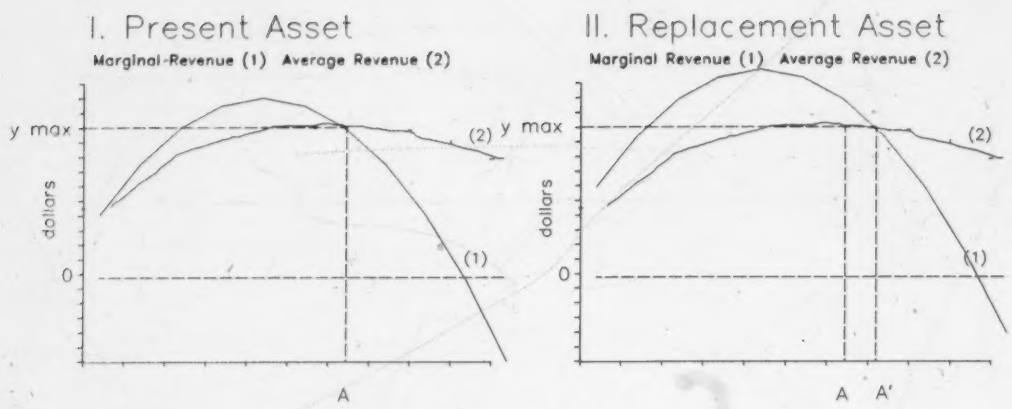

Figure 2.2. The optimum time for replacement of the present asset (A) determined by the opportunity cost of postponed replacement (y max) calculated from the marginal (1) and average net revenue curve (2) in case of non-identical replacement.[5]

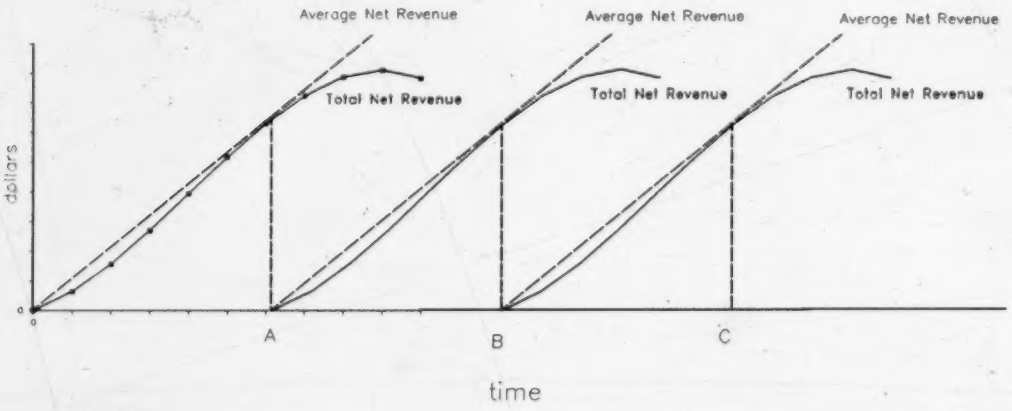

Figure 2.3 Determination of the optimal moment for replacing a dairy cow (A), (B), (C), in a continuous cycle. (Renkema and Stelwagen, 1979).[6]

5 adapted from Van Arendonk (1984). page 332.

6 Adapted from Dijkhuizen and Renkema. 1984. Page 267.

When time preference is considered, the optimum replacement time occurs when the marginal net revenue from the present asset is equal to the maximum annual annuity of anticipated net revenues from the subsequent assets. (Faris, 1960; Burt, 1965)

Perrin observed that the basic marginal principle is to compare gains from keeping the current asset for another time interval with the opportunity gains which could be realized from a replacement asset during the same period.

Thus an asset should be kept for an additional year if the additional net returns are greater than the "average" annual returns from a new asset.

The concept of the "average" opportunity gains derived from the replacement asset is confounded in that it involves the annualization of a lumpy stream of gains that are dependent upon the replacement policy for the replacement asset.

Perrin (1972) submitted that a higher discount rate could result in either earlier or later replacement depending on the shape of the marginal net revenue curve.

The capital asset replacement problem is best addressed through the use of marginal analysis. The solution of the problem is achieved by comparing the gains (dollars) produced by keeping the existing asset for another time period with gains (dollars) that could be produced by the replacement asset during the identical time period.

While the concept appears simple, the solution of the replacement problem is complex. In order to calculate the opportunity gains flowing from the replacement asset it is necessary to annualize the value of the net returns which is in turn dependent upon the specific replacement policy for the replacement asset that is utilized. It is, therefore, necessary to look at a continuum of time over which replacement occurs and to maximize the present value of the perpetual annuity which is produced by the numerous replacement assets over the time horizon under consideration. In order to convert the perpetual annuity into the required marginal cost (continuous) that is equivalent to an opportunity flow of residual earnings it is necessary to multiply the perpetual annuity by the "capital recovery factor."

Faris (1960) states that when considering assets with a long production period and with revenues being realized throughout the life of the asset, the principle of optimum replacement can be stated as:

> *"the optimum time to replace is when the marginal net revenue from the present enterprise is equal to the highest amortized present value of anticipated net revenues from the enterprise immediately following."*

In the analysis of dairy cows the annual net revenue is synonymous with the marginal net revenue. Thus, for dairy cattle, the optimum replacement age would be:

"when the annual net revenue from the present cow is equal to the expected amortized present value of the net revenue from the replacement cow."

The expected net revenues occur over the time horizon of the project. It is, therefore, necessary to evaluate the net revenues at a common point in time, specifically, the present time. Thus, the present value of the expected net revenues from the current and replacement assets must be calculated.

The asset manager's objective is to maximize the flow of net revenue over time. Thus, it is necessary to convert the present value of the net revenue in year "n" into an equivalent flow of income for the replacement asset. This allows the comparison of current marginal net revenue with the annual net revenue of the replacement asset. The use of the sinking fund formula:

$$\left[\frac{FV}{((1+i)^n-1)/i}\right]$$

converts the annuity value into an amortized present value of expected net revenue (Faris, 1969).

Faris and Perrin have introduced some confusion into the discussion by approaching the problem in a future value rather than a present value context. The use of the sinking fund to convert annuity values into an amortized present value is misleading in that the theory of the sinking fund derives from the question of the amount of funds that must be set aside at the end of each period to accumulate a lump sum of funds at some future point in time.

$$A = FV\left[\frac{i}{(1+i)^n-1}\right]$$

where

$$FV = \sum_{k=1}^{n} V_k(1+i)^k$$

The amortization theory derives from the question of the amount of funds that must be paid at the end of each period to discharge a known present value.

As calculated by:

$$A = PV\left[\frac{i}{1-\frac{1}{(1+i)^n}}\right] = PV\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$$

Since present value is the discounted future value

$$PV = \frac{FV}{(1+i)^n}$$

It follows that:

$$A = PV\left[\frac{i}{1-\frac{1}{(1+i)^n}}\right] = \frac{FV}{(1+i)^n}\left[\frac{i(1+i)^n}{(1+i)^n-1}\right] = FV\left[\frac{i}{(1+i)^n-1}\right]$$

Thus the results are the same, but the sinking fund notion is misleading as used by the aforementioned authors.

Burt (1965) incorporated involuntary replacement as the result of chance events into replacement theory. This enhancement to optimal replacement theory accounted for the reality that assets are subject to chance failure or destruction that requires the replacement of the asset in order to continue with the productive process. Dairy cows can be subjected to random replacement for a variety of reasons including disease and health disorders, injury or ultimately, death.

When the probability of involuntary replacement is included, the opportunity cost of postponed replacement is equal to the maximum annuity of the net revenues anticipated from the replacement assets. Before the annuity value can be determined, the present value of the future net revenues must be calculated. To calculate the present value, the future net revenues of each period are discounted and weighted for the probability of involuntary disposal.

Thus, the annuity factor used to convert the present value into an annuity (constant amount per time period) is a function of the discount rate AND of the probabilities of occurrence (Burt, 1965).

Burt (1965) showed that "*a positive chance of involuntary replacement (1-p) alters the optimal replacement policy of a deterministic model by merely increasing the discount rate.*"

As Faris (1960) has noted "costs and revenues can be increased or decreased by a constant amount each year without changing the optimum replacement pattern."

The reason for this characteristic of optimum replacement theory is that the formula for the present value of an annuity is the inverse of the formula for the amortization of a sum. Because the two formulas are inverses, they cancel each other out leaving only the constant.[7]

As a result of this characteristic, fixed costs can be ignored in the calculation of costs for the enterprise with no significant changes to the optimal replacement timing. In addition, as Faris notes, that fixed costs can be allowed to vary while holding other inputs and outputs constant and obtain

---

[7] $k\left[\frac{(1+r)^n-1}{r(1+r)^n}\right]\left[\frac{r(1+r)^n}{(1+r)^n-1}\right] = k$
where the first term is the present value of an annuity while the second term is the amortization formula.

the same optimal replacement policies. Thus, operators with major differences in fixed costs but experiencing more or less identical variable costs and returns would employ the same replacement strategies.

B. Dairy Studies

A number of studies into the dairy cow replacement problem have been completed in the past 25 years. The earlier work of Jenkins and Halter (1964) set the stage for a number of subsequent studies including Giaver (1966), Smith (1968), Redman and Kuo (1969), Stewart (1975), Gartner (1980), and more recently Van Arendonk (1984, 1985a, 1985b, 1986, 1987, 1988) and Van Arendonk and Dijkhuizen (1985).

The Jenkins and Halter's study was the first to explicitly deal with the problem of dairy cow replacement from a point of maximization of profit. The major objective of their work was to present the detailed development of a model to solve a multistage stochastic replacement decision problem. They used the problem of the replacement of dairy cows to illustrate their model which was solved by the use of dynamic programming techniques for a number of combinations of prices.

Giaver's 1965 study analyzed the problem of when, and/or under which conditions, to replace a dairy cow. The study developed an economic decision model utilizing dynamic programming with a value iterative algorithm and Markov processes.

Smith (1968) used a dynamic programming model to evaluate the dairy cow replacement problem. This model was then compared to a forward total enumeration approach. Smith developed a generalized prediction equation for milk yield using least squares regression techniques using the length of the previous calving interval and production in the two most recent lactations as the independent variables. Smith's study took into account genetic improvement in the replacement heifers but emphasized milk production as the major component of the replacement problem.

The Redman and Kuo study of 1969 was essentially an updated version of the Jenkins and Halter approach that incorporated a discounting procedure into the model.

McArthur (1973) developed a stochastic dynamic programming model for use as a culling decision aid. McArthur's model did not include changes in feed costs or values for beef. In fact, the only means of differentiation between cows in the same lactation was the average yield of butterfat.

Pearson and Freeman (1973) utilized deterministic models to evaluate four cow culling systems effect on profitability. These authors noted that while the most important economic trait of dairy cows in the United States is milk yield, that other traits have significant economic effects, but that their effect, from a genetic standpoint, is severely limited whether by low heritability or by a lack of objective measurement of the trait. It was concluded that the average age at calving determines the rate at which genetic improvement in the preceding generation is incorporated into the herd with the generational interval becoming more important as the rate of genetic improvement in sires increases.

Stewart employed a dynamic programming model to optimize dairy cow culling procedure. The study included milk returns, beef sales, feed costs, replacement costs and cow depreciation costs over a ten year period. Genetic improvement was included in the form of a 45 kgs. per year increase in milk yield. The results of Stewart's study are generally in agreement with the results of earlier studies (Jenkins and Halter, 1963; Redman and Kuo, 1969; Glaver, 1965; McArthur, 1973; and Smith, 1971). By the author's own admission, the study (as is the case with all the previous studies) is weakened due to the lack of adequate data (information) on many of the economic and biological relationships and their interrelationships.

Specifically lacking was data on the relationship between market value and biological traits such as age, milk yield, butterfat content and body weight. A more objective calculation of salvage values, cow depreciation and replacement costs would result in a more concise evaluation of market value. Better data on the relationship between age and body weight would result in less prediction error in the replacement model because of the effect of body weight on feed costs, salvage values and returns for beef sales. The lack of concise information on the relationship between veterinary and drug costs with age or milk production characteristics confounds the accurate determination of replacement policies.

Stewart, Burnside, and Pfeiffer in their 1978 article extrapolated on Stewart's 1975 study to encompass the four major dairy breeds rather than just Holsteins. A probability distribution of possible replacement heifers was incorporated into the model and the effects of replacement selection on the replacement decision were analyzed. The inclusion of the probability distribution of replacement heifers resulted in more intensive culling under all prices.

Stewart et. al. (1978) concluded that the use of an "average" replacement heifer resulted in underestimation of the value of the replacement heifer. When variation in production and body weight of the replacement heifers are included, rather than using average characteristics, more intensive culling resulted. Stewart also submitted that the value of the replacement was overestimated because all cows were assumed to remain in the same production and weight class throughout their entire life.

Renkema and Stelwagen (1979) utilized the underlying economic criterion that a cow of a particular age should be kept in the herd as long as her marginal profit (defined as the profit in one particular year, the change in slaughter value included) is higher than the expected average profit per year during a young replacement cow's life. Opportunity costs were considered because profits from a replacement cow can not be realized as long as the original herd cow is retained.[8] These author's noted that genetic increases in milk production only affect the expected income of the replacement cow. It was also contended that by using stochastic variables practical real life conditions can be very closely imitated.

Gardner (1979, 1980) developed a computer simulation model to examine the effects of different replacement rates on genetic improvement in yield per cow, milk output, farm profit and practical husbandry. It was observed that a major effect of an increased replacement rate was the reduced number of cows that could be carried on a given unit due to the increasing proportion of the unit utilized for the rearing of replacements.

Gardner, in his 1981 report observed the need to consider the time value of money in the calculation of replacement policies. He also noted that veterinary costs may increase as the maturity of the herd increases. Gardner (1982a), noted the importance of incorporating the estimates of survivability probabilities into the determination of replacement policies. A sinking fund calculation was used to determine the profitability of a particular replacement strategy. Various strategies were examined to determine the particular strategy that resulted in the largest value of the sinking fund at the conclusion of a given planning horizon.

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8 Marginal Earned Income defined as:
-for lactation 1: Net margin plus slaughter value at the end of lactation 1 minus the cost of the replacement heifer.
-for all other lactations: net margin plus slaughter value at the end of the particular lactation minus the slaughter value at the end of the immediately preceding lactation or the change in slaughter value over the period of the lactation.

Van Arendonk's detailed and exhaustive work on the replacement question has utilized dynamic programming models throughout to determine optimal replacement policies for dairy cattle. The objective utilized was to maximize the present value of net revenues of the existing cow and all her replacements over a planning horizon of 20 years.

The optimum replacement decisions determined corresponded with an average herd life of 42.9 months (3.575 years) for a replacement rate of 28% per year with the optimum policies significantly altered by changes in the cost of a replacement heifer or the salvage value of culled cows. Changes in the price of milk, calves or feed did not significantly affect the optimum herd life of cows. The production level of the herd or the rate of genetic improvement did not affect the optimum herd life of cows to a significant degree.

The model included an optimizing technique, a procedure for determining revenues and costs of dairy cows, as well as the marginal probabilities of involuntary disposal. Van Arendonk concludes that from an economic point of view, management and breeding policies should be directed towards a reduction in the involuntary disposal rate rather than emphasizing maximization of the average herd life of dairy cows.

Van Arendonk has studied the effect of various factors on optimal replacement policies. He has examined the "Influence of Variations in Reproduction and Production" (1985) wherein he noted no significant changes to the optimum policy from that of the previous work.

Seasonal variation was included in the 1986 study with the conclusion that heifers calving from September to November had the highest income. A significant affect by seasonal variation on the replacement policy was noted. The 1987 work concluded that the optimal replacement policy is significantly altered by the inclusion of variation in feed costs and carcass value.

Van Arendonk's work constitutes an accurate summary of the work to date on the derivation of optimal replacement strategies for dairy cows. His work, as noted, is the most detailed and exhaustive of the works to date.

C. The Proposed Study

The intent of this research project is to develop (derive) an economic model that predicts the economic replacement strategies that will maximize the dairy producers profit over an infinite time

horizon. Particular attention will be paid to the inclusion of genetic improvement factors and the question of longevity of the dairy cow as these factors relate to the overall profitability of the dairy cow

While the replacement question will be examined with a deterministic model, it is the intent of the research to include a comprehensive view of the variable factors that go into the production of milk and the annual maintenance of the dairy cow. Veterinary costs will be included in the model. A variety of scenarios will be developed and examined to evaluate the effect that fluctuating costs and returns have on replacement strategies (i.e. sensitivity analysis).

III. The Analytical Model

A. Introduction

This study will consider the replacement problem only within the context of profit-maximizing behavior. As such, the problem is to search for a net present value maximizing replacement strategy.

In the following discussions, it will be assumed that:

a) replacement will be defined as the replacement of an existing cow with a genetically superior cow.

b) all cashflows are received at the end of the lactation under consideration.

c) all replacement cows are purchased under the assumption that the rearing of ones own replacements is a separate economic activity.

d) the replacement cow is purchased at the beginning of the lactation.

e) the replacement cow has a calf and commences milk production on the day of purchase.

f) only integer values of the lactations are available for consideration.

g) the maximum productive life of the dairy cow is ten lactations.

The economic question to be modelled can be expressed as: "Does one more year in a repeating cycle cause the annuity value to rise, fall, or remain the same."

The situation can be represented mathematically as a comparison of:

$$\left[\frac{R_1}{(1+i)^1}+\ldots+\frac{R_n}{(1+i)^n}\right]\left[\frac{i}{1-\frac{1}{(1+i)^n}}\right] \qquad (1)$$

where R_n = the net cashflow associated with period "n".

for the first "n" periods to:

$$\left[\frac{R_1}{(1+i)^1}+\ldots+\frac{R_n}{(1+i)^n}+\frac{R_{n+1}}{(1+i)^{n+1}}\right]\left[\frac{i}{1-\frac{1}{(1+i)^{n+1}}}\right] \qquad (2)$$

for the first "n" periods plus one additional period.

Thus, the decision rules based on the principle of marginal returns equal to marginal costs (opportunity cost of keeping the existing asset one more period) can be expressed as a comparison of:

$$\left[\sum_{t=1}^{n}\frac{R_t}{(1+i)^t}\right]\left[\frac{i}{1-\frac{1}{(1+i)^n}}\right] \quad to \quad R_{n+1} \tag{3}$$

If the average of the first "n" periods exceeds the amount in the next period, then the asset should be replaced. If it does exceed the amount in the next period, then the asset should be kept for one more period.

As long as the net present value is increasing from one period to the next, the decision rule states that the asset should be retained for the next period.

If the net present value decreases from one period to the next, then decision rule states that the asset should be replaced with a new one of similar characteristics for the next period.

If the net present value is constant, then one is indifferent between keeping or replacing the asset.

B. Net Present Value Components

The replacement issue has a number of components, including the milk production component, the calf born at the onset of each lactation cycle, and the initial purchase of the replacement heifer and the subsequent sale or salvage of the cow at the end of the cycle.

1. The Milk Component and Genetic Growth

The first step in the development of the model is to determine the present value of the net cash flows associated with the milk production of the first cow in the replacement cycle.

Where:

V_t = the annual net cash flow from milk production associated with a specific cow in lactation "t".

i = the discount rate.

n = the number of lactations for which the cow is kept.

The present value of the milk component of the first cow can be expressed as:

$$\sum_{t=1}^{n}\frac{V_t}{(1+i)^t} \tag{4}$$

This cow will be replaced at the end of "n" periods by a cow that is genetically superior by a factor of $(1+g)^n$. This is based on the notion that there is "g%" growth in the genetic pool per period and "n" periods have elapsed since the start of the first cow.

The present value of the second cycle will then be:

$$\left[\sum_{t=1}^{n}\frac{V_t}{(1+i)^t}\right]\left[\frac{(1+g)^n}{(1+i)^n}\right] \tag{5}$$

Where:

g = the rate of genetic improvement occurring per year expressed as a percentage increase in milk yield per year per cow.

The first cycle has grown by the factor "g" per lactation over the "n" periods of the first cycle but it starts "n" periods hence and so must be brought back to the present. By similar reasoning, the present value of cycle "k" is:

$$\left[\sum_{t=1}^{n}\frac{V_t}{(1+i)^t}\right]\left[\frac{(1+g)^{n(k-1)}}{(1+i)^{n(k-1)}}\right] \tag{6}$$

The sum of the present values of all future cycles can then be expressed as:

$$NPV_{milk} = \left[\sum_{t=1}^{n}\frac{V_t}{(1+i)^t}\right]\left[1+\frac{(1+g)^n}{(1+i)^n}+\frac{(1+g)^{2n}}{(1+i)^{2n}}+\frac{(1+g)^{3n}}{(1+i)^{3n}}+\ldots+\frac{(1+g)^{(k-1)n}}{(1+i)^{(k-1)n}}\right]. \tag{7}$$

If this process is repeated for an infinite number of cycles (i.e. as "k" approaches infinity), then the net present value is:

$$NPV_{milk} = \left[\sum_{t=1}^{n}\frac{V_t}{(1+i)^t}\right]\left[\frac{(1+i)^n}{(1+i)^n-(1+g)^n}\right] \tag{8}$$

2. The Calf and Capital Component

The capital component consists of the initial cost of the cow and her subsequent salvage value. For convenience, the sale of the calf will be included with the capital component of the model. It is assumed that there is no growth to either component. This assumption rests on the premise of an efficient market.

The present value of the first cycle is:

$$NPV_{capital} = \left[-C+\frac{SV_n}{(1+i)^n}+\sum_{t=1}^{n}\frac{B_t}{(1+i)^t}\right] \tag{9}$$

Where

C = the cost of the replacement heifer,

SV_n = the salvage value of the existing cow in period "n", and

B_t = the value of the calf in period "t".

It follows that the present value of the second cycle is:

$$NPV_{capital} = \left[-C + \frac{SV_n}{(1+i)^n} + \sum_{t=1}^{n} \frac{B_t}{(1+i)^t} \right] \left[\frac{1}{(1+i)^n} \right] \tag{10}$$

and that the present value for cycle "k" is:

$$NPV_{capital} = \left[-C + \frac{SV_n}{(1+i)^n} + \sum_{t=1}^{n} \frac{B_t}{(1+i)^t} \right] \left[\frac{1}{(1+i)^{(k-1)n}} \right] \tag{11}$$

The sum of all future cycles can then be expressed as:

$$NPV_{capital} = \left[-C + \frac{SV_n}{(1+i)^n} + \sum_{t=1}^{n} \frac{B_t}{(1+i)^t} \right] \left[1 + \frac{1}{(1+i)^n} + \frac{1}{(1+i)^{2n}} + \frac{1}{(1+i)^{3n}} + \ldots + \frac{1}{(1+i)^{(k-1)n}} \right]. \tag{12}$$

Which in the limit reduces to:

$$NPV_{capital} = \left[-C + \frac{SV_n}{(1+i)^n} + \sum_{t=1}^{n} \frac{B_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - 1} \right] \tag{13}$$

3. The Milk and Capital Components Combined

Combining the two components into one equation results in:[9]

$$NPV_n = \left[\left\{ \left[-C + \frac{SV_n}{(1+i)^n} + \sum_{t=1}^{n} \frac{B_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - 1} \right] + \left[\sum_{t=1}^{n} \frac{V_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - (1+g)^n} \right] \right\} \right] \tag{14}$$

If there was no involuntary culling of cows, (i.e. all cows would continue through to period "n"), then one would be justified in directly invoking the decision rule coming from

$$NPV^* = \max_{n=1}^{N} [NPV_n] \tag{15}$$

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9 Equation 14 can be expressed on an annual basis as:

$$ANPV_n = \left[\left\{ \left[-C + \frac{SV_n}{(1+i)^n} + \sum_{t=1}^{n} \frac{B_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - 1} \right] + \left[\sum_{t=1}^{n} \frac{V_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - (1+g)^n} \right] \right\} (i) \right]$$

(16)

$$NPV^{*} = \underset{n=1}{\overset{N}{\text{Max}}}\left[\left[-C + \frac{SV_n}{(1+i)^n} + \sum_{t=1}^{n}\frac{B_t}{(1+i)^t}\right]\left[\frac{(1+i)^n}{(1+i)^n - 1}\right] + \left[\sum_{t=1}^{n}\frac{A_t}{(1+i)^t}\right]\left[\frac{(1+i)^n}{(1+i)^n - (1+g)^n}\right]\right]$$

If NPV is maximum, then keeping the cow for one more period will cause a decrease in NPV. That is to say that:

$$NPV_n > NPV_{n+1}. \tag{17}$$

In the real world, cows have some probability of involuntary removal from the herd. For this reason, equation (16) must be modified to reflect the concept that it is only an intermediate step in the process of the solution to determine an economic replacement strategy. Equation (18), is the mathematical representation of the net present value incorporating both operating and capital flows for all cycles from one through "N" periods of "n" length:

(18)

$$NPV_n = \left[\left\{\left[-C + \frac{SV_n}{(1+i)^n} + \sum_{t=1}^{n}\frac{B_t}{(1+i)^t}\right]\left[\frac{(1+i)^n}{(1+i)^n - 1}\right] + \left[\sum_{t=1}^{n}\frac{A_t}{(1+i)^t}\right]\left[\frac{(1+i)^n}{(1+i)^n - (1+g)^n}\right]\right\}\right]$$

This results in a vector of net present values:

$$[NPV] = \begin{bmatrix} NPV_1 \\ NPV_2 \\ \cdots \\ \cdots \\ NPV_N \end{bmatrix} \tag{19}$$

C. Incorporating Involuntary Disposal

1 Involuntary Disposals

The development to this point assumes that the cow will continue in production through "n" periods. In reality, the cow may fail to continue through the complete cycle. This means that with each lactation there is a probability that the lactation will be completed and that the cow will proceed to the next lactation. A cow may fail to advance to the next lactation for any number of reasons including death, health disorders, or failure to conceive.

The situation can be described as a discrete - finite Markov process (Hillier and Lieberman, 1986; Agrawal and Heady, 1972). It is finite because there is a finite number of states (lactations) possible and it is discrete because only integer states (lactations) are to be considered. Thus, the progression of a dairy cow from one lactation to the next can be represented in the form of a transition probability matrix T with elements T_{ij}, $i = 1, 2, \ldots n$; $j = 1, 2, \ldots n$.

Thus, the appropriate Markov process can be represented by:

$$\begin{bmatrix} d_1 & d_2 & d_3 & \ldots & d_{n-1} & 1 \\ 1-d_1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 1-d_2 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1-d_3 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 1-d_{n-1} & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 \end{bmatrix} \begin{bmatrix} C_{k,1} \\ C_{k,2} \\ C_{k,3} \\ \ldots \\ C_{k,n-1} \\ C_{k,n} \end{bmatrix} = \begin{bmatrix} C_{k+1,1} \\ C_{k+1,2} \\ C_{k+1,3} \\ \ldots \\ C_{k+1,n-1} \\ C_{k+1,n} \end{bmatrix} \qquad (20)$$

With $[C_{k,1} + C_{k,2} + C_{k,3} + \ldots + C_{k,n} = 1]$

where matrix T is post multiplied by vector $C_{k,n}$ containing the herd composition in the current period to obtain the herd composition in the subsequent time period $C_{k+1,n}$

Of necessity, the herd composition vector must be exhaustive (i.e. account for 100% of the cows - sum to 1)

The cells in the first row of the matrix T represent the transition probabilities of a cow failing involuntarily in the specific lactation and being replaced by another cow. The cell $T_{2,1}$ in the succeeding row of the matrix T represents the probability $(1 - d_1)$ that the cow will in be state 2 in the next period (i.e., that the cow will not be culled involuntarily). For the cow in lactation 2 there is a probability $(1 - d_2)$ that she will advance to lactation 3 in the next period and a probability d_2 that she will be involuntarily culled and replaced with a cow that is in lactation 1. If a cow is in lactation $n - 1$ then there is a probability $(1 - d_{n-1})$ that she will advance to state "n" and a probability d_{n-1} that she will be culled. A cow in lactation "n" will be culled with certainty. C_j represents the proportion of cows in lactation "j"

Solution of the system in steady state results in:

(21)

$$\begin{bmatrix} C_{k,1} \\ C_{k,2} \\ C_{k,3} \\ \cdots \\ C_{k,n-1} \\ C_{k,n} \end{bmatrix} = \begin{bmatrix} C_{k+1,1} \\ C_{k+1,2} \\ C_{k+1,3} \\ \cdots \\ C_{k+1,n-1} \\ C_{k+1,n} \end{bmatrix}$$

or

$$TC_s = C_s. \tag{22}$$

Where C_s is the steady state vector representing herd composition.

Conditions for Steady State to Occur

$$1)\,[T][C_s] = [C_s] \tag{23}$$

$$2)\,\sum_{j=1}^{N} C_j = 1$$

For example, in a two lactation pattern:

$$\begin{bmatrix} d_1 & 1 \\ 1-d_1 & 0 \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \tag{24}$$

$$C_1 + C_2 = 1$$

Multiplication results in three equations and two unknowns:

$$\begin{aligned} C_1 d_1 + C_2 &= C_1 \\ (1-d_1)C_1 + 0 &= C_2 \\ C_1 + C_2 &= 1 \end{aligned} \tag{25}$$

This system of equations can be solved using the following two equations; the number of cows entering the second lactation and the total number of cows in lactation one and two.

$$\begin{aligned} C_1 + C_2 &= 1 \\ (1-d_1)C_1 - C_2 &= 0 \end{aligned} \tag{26}$$

This can be expressed as:

$$\begin{bmatrix} 1 & 1 \\ (1-d_1) & -1 \end{bmatrix}\begin{bmatrix} C_{s,1} \\ C_{s,2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{27}$$

And solved by inversion as follows:

$$\begin{bmatrix} 1 & 1 \\ (1-d_1) & -1 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} C_{s,1} \\ C_{s,2} \end{bmatrix} \tag{28}$$

In general, one can create the adjusted matrix T_a from the matrix T and solve the system:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & -1 \\ 1-d_1 & -1 & 0 & 0 & 0 \\ 0 & 1-d_2 & -1 & 0 & 0 \\ 0 & 0 & 1-d_3 & -1 & 0 \\ 0 & 0 & 0 & 1-d_{n-1} & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} C_{s,1} \\ C_{s,2} \\ C_{s,3} \\ C_{s,n-1} \\ C_{s,n} \end{bmatrix} \qquad (29)$$

2. Replacement Patterns

By examining the within the herd composition, it can be concluded that those cows in the herd at their "n"th lactation are being recycled in a "n" lactation period. This means that there must be C_n cows in this pattern in each lactation up to and including the "n"th lactation. For example, if there are ten cows in the herd in their fifth lactation and there is a replacement strategy that dictates all cows are replaced after five lactations, it must be so that there are fifty cows in the herd predestined to be replaced after their fifth lactation. Of the cows (suppose for example there are sixteen of them) in their fourth lactation, there will be included those cows predestined to go on to their fifth lactation, (i.e. ten will go on to their fifth lactation, while six will be involuntarily culled after the fourth lactation). Thus, there must be 24 cows (4 x 6) to support this pattern. The cows in the remaining patterns can be calculated in similar fashion.

In general, the solution to the Markov system can be portrayed in matrix form as $C_1 > C_2 > C_3 > \ldots > C_{n-1} > C_n$.[10] In general if there are C_n cows in a pattern that recycles in "n" periods, then there must be C_n in each lactation stage to feed this cycle. There must, as a result be nC_n cows in the total system. If there are C_n cows in state n there must be C_{n-1} in state n-1 but C_n of these cows are already part of the "n" cycle pattern. Therefore, there are $C_{n-1} - C_n$ cows in the n-1 cycle at each stage in the system. The total number of such cows, predestined to be replaced in n-1 years is then $(n-1)(C_{n-1} - C_n)$. This pattern will continue until the last state where $1(C_2 - C_1)$. The cows in each pattern will then be:

10 To reduce notational complexity, the subscript "s" will be dropped on the understanding that from here on the vector C is in steady state.

$$\begin{bmatrix} 1(C_1 - C_2) \\ 2(C_2 - C_3) \\ \cdots \\ (n-1)(C_{n-1} - C_n) \\ nC_n \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ \cdots \\ Q_{n-1} \\ Q_n \end{bmatrix} \qquad (30)$$

where vector Q represents the number of cows in each pattern.

Having determined the proportion of the herd in various patterns of replacement, it is possible to determine the average net present value of the herd. This can be accomplished by weighting the net present values computed as the vector NPV_n in equation (19)

D. The Objective Function

We can then weight the net present value from each feasible cycle by these proportions "Q" to determine the weighted average net present value of the stall (herd).

Thus:

$$[Q_1 \quad Q_2 \quad Q_3 \quad Q_{n-1} \quad Q_n] \begin{bmatrix} NPV_1 \\ NPV_2 \\ NPV_3 \\ NPV_{n-1} \\ NPV_n \end{bmatrix} = NPV_n^* \qquad (31)$$

This procedure will allow the maximization of the WNPV function such that the maximum value of the weighted average NPV of the stall can be determined. The multiplication of the WNPV by the discount rate (either on a per annum format or on a per lactation format) will give the annual or per lactation value of the associated perpetual cashflows.

The optimum replacement time is determined by finding the maximum weighted perpetual net present value:

$$WNPV_{n^*} = \underset{n=1}{\overset{N}{MAX}} \{Q' NPV\} \qquad (32)$$

where

Q' = the transposed vector of weighted proportions in the herd at each lactation,

NPV = the vector of perpetual net present values for cows kept for cycle "n", and

"n" = the optimum economic replacement cycle.

E. Flowchart of the Model

The procedure followed in analyzing and solving the problem is summarized by the following flowchart:

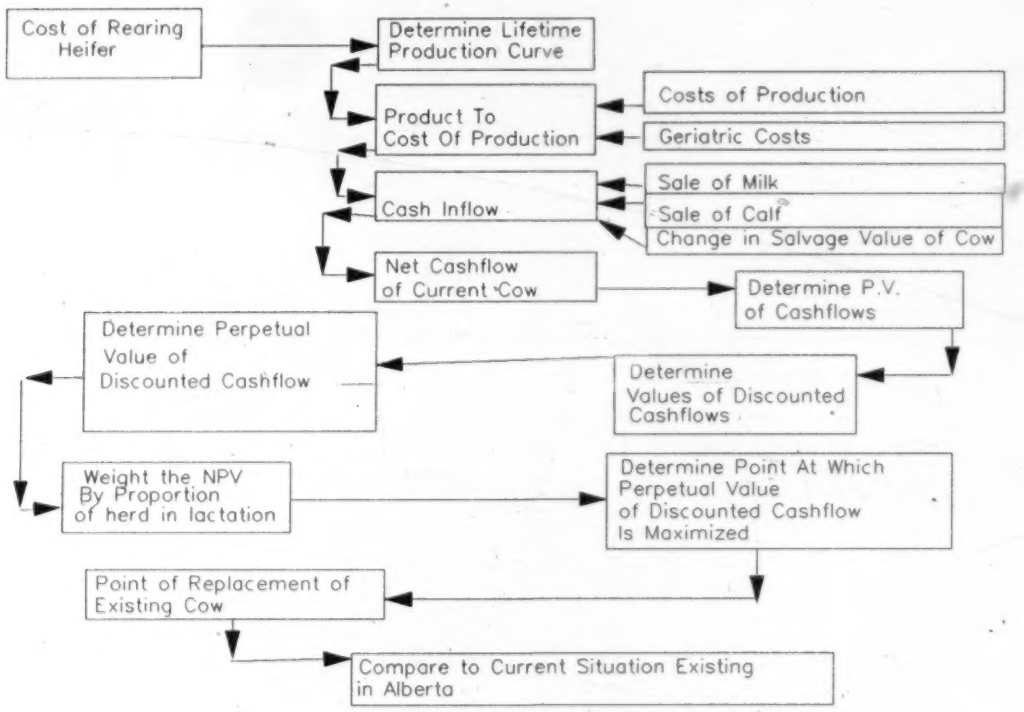
Cost of Rearing Heifer
Determine Lifetime Production Curve
Costs of Production
Product To Cost Of Production
Geriatric Costs
Sale of Milk
Cash Inflow
Sale of Calf
Change in Salvage Value of Cow
Net Cashflow of Current Cow
Determine P.V. of Cashflows
Determine Perpetual Value of Discounted Cashflow
Determine Values of Discounted Cashflows
Weight the NPV By Proportion of herd in lactation
Determine Point At Which Perpetual Value of Discounted Cashflow Is Maximized
Point of Replacement of Existing Cow
Compare to Current Situation Existing in Alberta

FIGURE 3.1 FLOWCHART FOR ECONOMIC REPLACEMENT PROJECT

F. Summary

The complete model by which the optimal replacement time will be calculated and the methodology for calculating this point in time have been put forth in the sections immediately preceding. Chapter four will now put empirical data into this analytical model.

IV. The Empirical Model

A. Lifetime Milk Yield Curve

A sample of 833 cows representing 2656 lactations was collected from four dairy producers.

An Ordinary Least Squares Regression was carried out on the data using the independent variables: Age, Calving Interval, Days In Milk, Butterfat, and a series of Dummy Variables to represent Lactation Number and a Constant while Milk Production was the dependent variable. The results of this OLS Regression are as indicated in Appendix I.

The Predicted Milk Yields by lactation from the OLS Regression, the Mean Yields and Standard Deviations associated with the samples are reported in Table 4.2.

The sample data used in determining the Regression Coefficients are biased in that the sample data have been selected by virtue of production and longevity. Only those cows with superior production and longevity characteristics are represented in the later lactation numbers. This is evidenced by the continuously declining number of records represented at each successive lactation (See Figure 4.3). As such, the sample data is prescreened and biased towards higher production in the latter lactations than would be the case with unselected data.

Two further OLS Regressions were carried out. The same independent and dependent variables were used but the sample was adjusted to include firstly, only those cows completing four or more lactations; and secondly, only those cows completing five or more lactations. The sample of cows completing four or more lactations was comprised of 1620 observations. The results are reported in Appendix II and the Predicted Milk Yields are reported in Table 4.1.

For the sample representing cows completing five or more lactations, a sample of 1219 observations was analyzed. The results are reported in Appendix III and the Predicted Milk Yields are reported in Table 4.1.

These predicted results are depicted graphically in Figure 4.1.

Of interest, is the observation that the cows with the greater longevity, produce initially at a lower level than the population in total and reach maximum production one lactation later (in both cases for 4 plus and 5 plus lactation cows) than the total population.

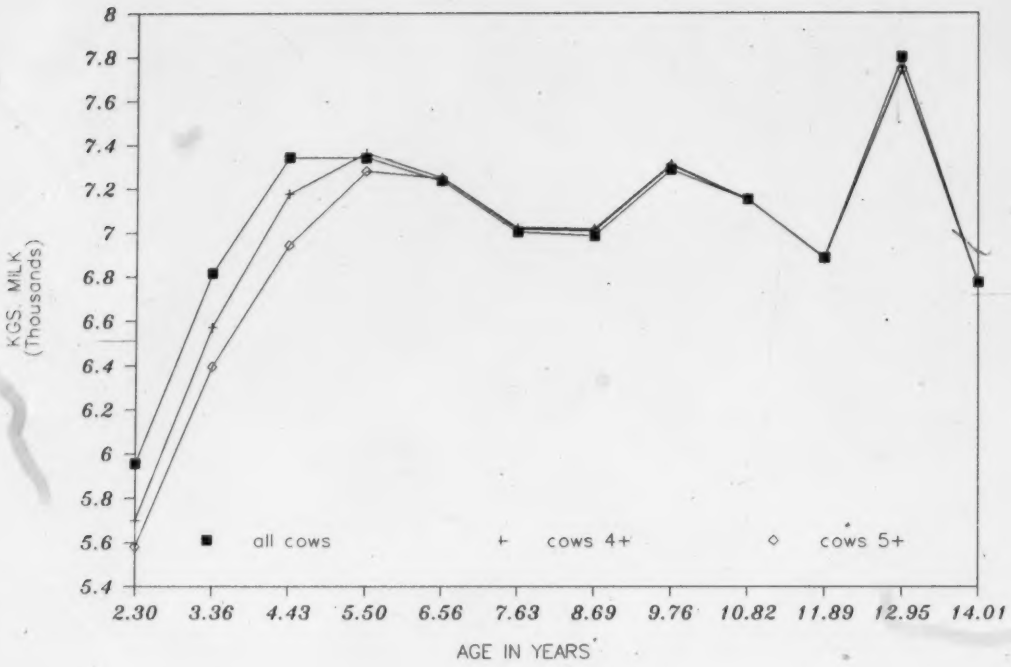

FIGURE 4.1 PREDICTED LIFETIME PRODUCTION

Because of the bias displayed in the latter lactations; the production for lactations eight, nine and ten were extrapolated from the predicted values determined in the OLS regressions by use of the age correction factors found in Table 4.4. The milk production levels used in the model are set forth in Table 4.1 and 4.3 and are displayed graphically in Figure 4.2.

The graphed values depicted in Figure 4.2 are set forth in column 2 of Table 4.1.

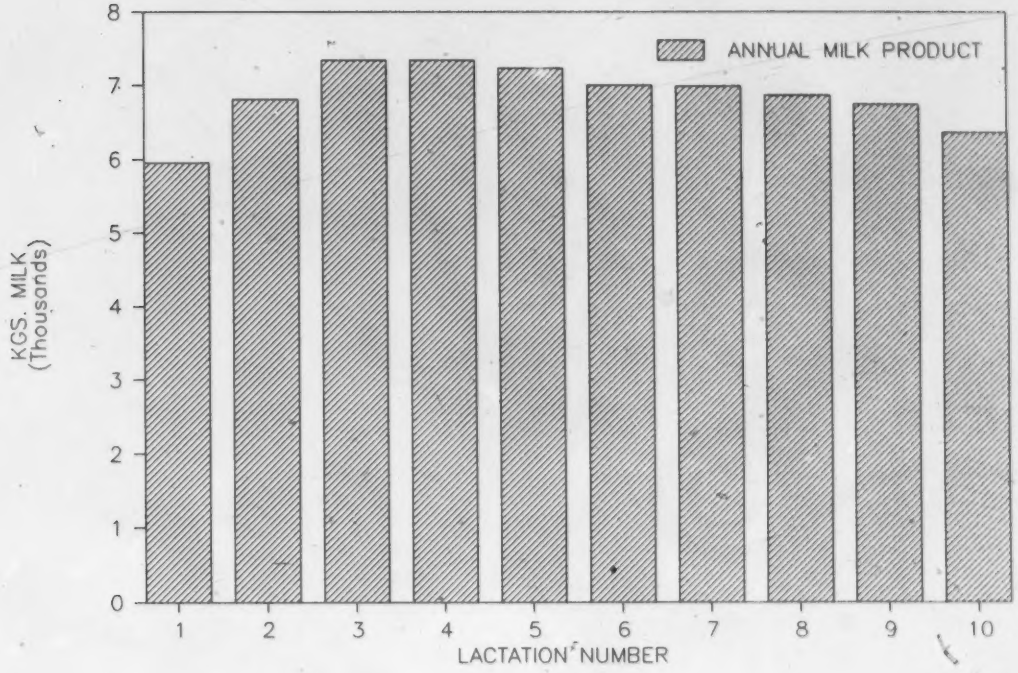

Figure 4.2 Milk Production By Lactation Used In Model

Table 4.1 Predicted Milk Yields

Lactation Number	All cows	4 + Lactations	5 + Lactations
1	5954	5699	5582
2	6815	6573	6396
3	7341	7179	6943
4	7340	7363	7283
5	7235	7252	7251
6	7001	7023	7014
7	6983	7019	7010
8	6864	7313	7306
9	6740	7156	7152
10	6365	6884	6882

Column 2 of Table 4.2 sets forth the mean milk production while column 3 sets forth the standard deviation in milk production for each lactation. Column 4 sets forth the predicted milk production for each lactation using the OLS coefficients. Column 5 sets forth the predicted milk production for each lactation using the age correction factors of Table 4.4 applied to the maximum predicted milk production from column 4.

Table 4.2 Production Mean, and Standard Deviations of Sample Data				
Lactation Number	Mean Product.	Standard Deviation	Predicted Product. (regress.)	Predicted Production (age-correct)
1	6214	1871	5954	5982
2	6730	1804	6815	6674
3	7188	1943	7341	7341
4	7290	1809	7340	7917
5	7095	1843	7235	8075
6	6977	1689	7001	8075
7	6992	2093	6983	7995
8	7057	2183	6864	7917
9	7407	1432	6740	7764
10	6611	2461	6365	7340
11	6373	2695		

Table 4.3 compares the milk production curves used by Van Arendonk (1985) and Beaudry and Cassel (1988) as well as the standard deviations of production from Beaudry and Cassel's (1988) predicted production. These values are compared to the predicted production used in this study in column 5.

Table 4.3 Lifetime Production Estimates[11]				
Lactation Number	Mean Milk[12] Production	Mean Milk[13] Production	Standard[14] Deviation	Milk[15] Production
1	4970	5945	1705	5954
2	5761	6969	1904	6815
3	6447	7425	2060	7341
4	6769	7336	2312	7340
5	6937	6788	2626	7235
6	6972	6536	2683	7001
7	7000	6260	2683	6983
8	6951	5835	2649	6250
9	6902	5445	2544	5700
10	6762	4428	2502	5000

Table 4.4 depicts the age correction factors to correct maximum production to various ages as adapted from Schmidt and Van Vleck (1974).

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11 Adapted from: a) Van Arendonk (1985).
b) Beaudry and Cassel (1988).

12 Van Arendonk (1985) Agricultural Systems, Vol. 16:180.

13 Beaudry and Cassell. (1988). Journal of Dairy Science, Vol. 71:206.

14 Beaudry and Cassel (1988) pp. 206.

15 As determined from Prediction equation using Regression coefficients calculated as previously described.

Table 4.4 Age Correction Factors[16]										
Lact	2	3	4	5	6	7	8	9	10	11
	1.35	1.22	1.10	1.02	1.00	1.00	1.00	1.01	1.02	1.04

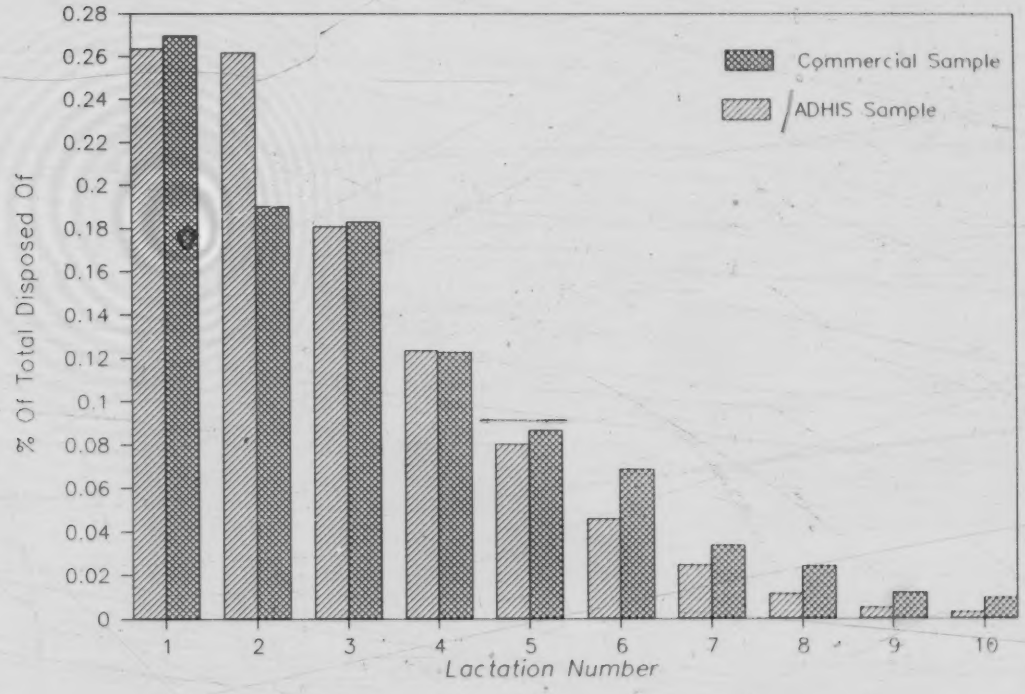


Figure 4.3. Cow Disposals By Lactations

The data that Figure 4.3 is derived from are set forth in Tables 4.5, 4.6. and 4.7. Table 4.5 indicates the frequency distribution of lactations from the collected data sample with the related descriptive statistics.

16 Adapted from Schmidt and Van Vleck (1974)

Table 4.5. Frequency Distribution of Lactations.						
Lact	N	Mean	St Dev	Variance	Minimum	Maximum
1	833	6214.1	1870.5	.34989E+07	475	14503
2	609	6729.7	1804.1	.32549E+07	583	12642
3	451	7188.4	1942.9	.37748E+07	388	13199
4	299	7289.7	1809.2	.32733E+07	1770	12925
5	197	7094.5	1842.9	.33962E+07	195	12124
6	125	6977.2	1688.7	.28518E+07	2510	13748
7	68	6992.3	2092.9	.43801E+07	2462	13950
8	40	7056.5	2183.0	.47654E+07	3000	11023
9	20	7406.7	1431.9	.20503E+07	4186	10355
10	10	6611.1	2460.9	.60561E+07	1064	10015
11	3	6372.7	2694.7	.72612E+07	3272	8148
12	1					

Table 4.6. Frequency Distribution Of Commercial Disposals.		
Commercial Herd Disposals		
Lactation Number	Number Of Disposals	Cumulative Disposals in %
1	224	26.89%
2	158	45.86%
3	152	64.11%
4	102	76.35%
5	72	84.99%
6	57	91.84%
7	28	95.20%
8	20	97.60%
9	10	98.80%
10	7	99.64%
11	2	99.88%
12	1	100.00%

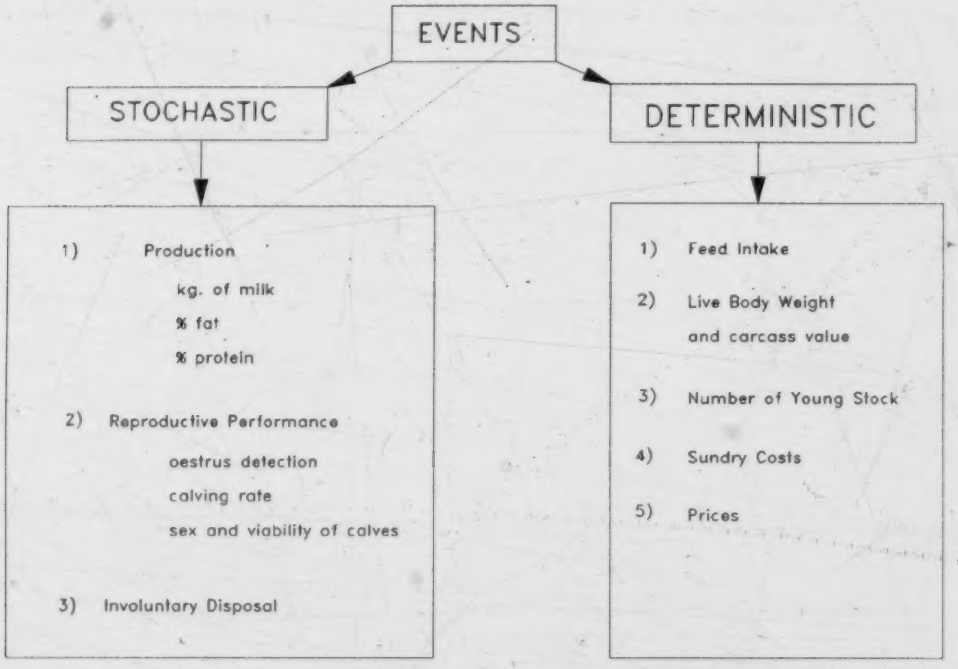
Table 4.6 sets forth the frequency distribution of lactations and the cumulative disposals of the collected commercial data sample as a percentage. Table 4.7 sets forth the same information for the data from the ADHIS data on cow disposals.

The two samples follow similar patterns but the ADHIS sample has 70.56% of the cows disposed of by the end of lactation three while the commercial sample has 64.11% of the cows disposed of by the same point. This is indicative of the cows in the commercial sample being kept longer than the cows in the ADHIS sample.

Table 4.7. Frequency Distribution Of ADHIS Disposals		
ADHIS Herd Disposals		
Lactation Number	Number Of Disposals	Cumulative Disposals as %
1	12329	26.34%
2	12238	52.50%
3	8456	70.56%
4	5776	82.90%
5	3760	90.95%
6	2153	95.55%
7	1157	98.02%
8	536	99.17%
9	240	99.68%
10+	150	100.00%

B. Determination Of Cash Flows For the Model

Many factors interact in the milk production process. The level of milk production is influenced by nutrition, age of the cow, size of the cow and the health of the cow. Whether the cow continues in the herd can be a factor of genetics, health, reproductive failure or even the failure (death) of the cow. These interactions are depicted in Figure 4.4.



adapted from Dijkuizen, Stelwagen, and Renkema (1987) page 400

FIGURE 4.4 SUMMARY OF FACTORS AFFECTING CASH FLOW[17]

The economic data included in the model are set forth in Table 4.8:

17 Van Arendonk (1988) in the Journal of Dairy Science. Vol.71:1051.

Table 4.8 Economic Data Used In The Base Case	
heifer cost/head	$1100
young cow $/kgs	$1.32
medium cow $/kgs	$1.10
canners and cutters $/kgs	$0.99
calf sale $/kgs	$1.54
Genetic Impovement/yr	0.50
milk price/$/hl	$49.11
Mastitis treatment $/yr	$20
veterinary costs $/yr	$20
Mastitis cost increase/yr	$5
veterinary cost inc/yr	$3
roughage cost $/kgs	$0.11
grain $/kgs	$0.25

Veterinary costs are included in the model. Mastitis treatment costs and the ensuing lost milk production and sales are a cost to the dairyman. These costs have predictable values. The mastitis costs increase with age and lactation. These facets of the milk production process and costs are portrayed in Table 4.9 and Table 4.10.

Table 4.9 sets forth the veterinary costs and value of unsalable milk associated with mastitis for each lactation. As indicated, there is a constant increase in the value of unsalable milk with each lactation as well as a constant increase in mastitis treatment costs per lactation.

Table 4.9 Veterinary Costs Associated With Mastitis[18]					
Lact Number	Value of Unsalable Milk	Increase per year of age	Treat costs	Increase in Treat. Costs/year	Total Costs /Year[19]
1	32.28	3.11	17.94	1.33	59.10
2	32.28	3.11	17.94	1.33	63.54
3	32.28	3.11	17.94	1.33	67.98
4	32.28	3.11	17.94	1.33	72.42
5	32.28	3.11	17.94	1.33	76.86
6	32.28	3.11	17.94	1.33	81.30
7	32.28	3.11	17.94	1.33	85.74
8	32.28	3.11	17.94	1.33	90.18
9	32.28	3.11	17.94	1.33	94.62
10	32.28	3.11	17.94	1.33	99.06

Table 4.10 Total Veterinary Costs			
Lactation Number	Mastitis Costs[20]	Other vet Costs[21]	Total Vet Costs
1	59.10	18.13	77.23
2	63.54	18.13	81.67
3	67.98	18.13	86.11
4	72.42	18.13	90.55
5	76.86	18.13	94.99
6	81.30	18.13	99.43
7	85.74	18.13	103.87
8	90.18	18.13	108.31
9	94.62	18.13	112.75
10	99.06	18.13	117.19

The physical data and setup of the model can be seen by examining Table 4.11 which immediately follows. Table 4.12 shows the cashflows from the model utilizing the base case data. Table 4.13 shows the NPV and ANPV of the perpetual replacement sequences with no involuntary replacement.

Table 4.14 shows the WNPV and WANPV of the perpetual replacement sequences incorporating involuntary replacement.

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18 adapted from:

a) Congleton. (1984). Journal of Dairy Science. Vol. 67:640

b) Hanson, Touchberry, Young, and Miller. (1979). Journal of Dairy Science, Vol. 62:1939.

19 Associated with Incidence of Mastitis only.

20 From Table 4.9.

21 Hanson, Touchberry, Young, and Miller. (1979). pp.1939.

Table 4.11 PHYSICAL DATA										
Lactation	1	2	3	4	5	6	7	8	9	10
cow wt	485	528	559	603	603	603	600	590	580	559
calf wt	50	50	50	50	50	50	50	50	50	50
milk prod	5954	6815	7341	7340	7235	7001	6983	6864	6740	6365
kg/day	20	22	24	24	24	23	23	23	22	21
hay kgs	5311	5782	6121	6603	6603	6603	6570	6461	6351	6121
grain kg	1952	2501	2898	2898	2898	2898	2898	2501	2501	2501

Table 4.12 CASHFLOWS FROM THE MODEL										
Lactation	1	2	3	4	5	6	7	8	9	10
milk sales	2837	3247	3498	3497	3447	3336	3327	3270	3211	3033
total $ in	2837	3247	3498	3497	3447	3336	3327	3270	3211	3033
feed costs	1072	1261	1398	1451	1451	1451	1447	1336	1324	1299
vet costs	91	99	107	115	123	131	139	147	155	163
misc costs	255	292	315	315	310	300	299	294	289	273
total $ out	1418	1652	1819	1880	1883	1881	1885	1777	1767	1734
surplus	1419	1595	1679	1617	1564	1454	1442	1494	1444	1299
salvage $	640	697	738	663	663	663	594	584	574	553
calf sales	77	77	77	77	77	77	77	77	77	77
RESIDUAL	2136	2369	2494	2358	2304	2195	2113	2155	2095	1929

Table 4.13 NPV and ANPV Without Involuntary Replacement										
Lactation	1	2	3	4	5	6	7	8	9	10
NPV	11351	14000	14157	13261	12360	11354	10374	9563	8789	8021
ANPV	962	1187	1200	1124	1048	962	879	811	745	680

Table 4.14 WANPV and WNPV WITH INVOLUNTARY REPLACEMENT										
Lactation	1	2	3	4	5	6	7	8	9	10
WNPV	11351	13825	14011	13331	12674	11998	11387	10915	10505	10145
WANPV	962	1172	1188	1130	1074	1017	965	925	890	860

C. Choice Of A Discount Rate

A discount rate reflects the relative value of future earnings versus the present earnings of an asset and can, thus be thought of as the relative time value placed on money. Furthermore, the discount rate can be used to account for the inherent risk in the investment (project).

For the purposes of this research project, the discount rate will be based on the Market Rate of Return on Investments of Similar Risk approach (Andrews, 1987. page 12.) wherein the appropriate discount rate will be based on a base risk-free rate[22] plus a risk premium that is based on the degree of risk experienced by the investor (the dairy operator) in comparison to the market. Since all flows of cash used in this study are measured in real terms, the discount rate chosen will also be an inflation-free rate.

In Alberta, Fluid Milk is priced by a formula that includes the following components in the relative proportions indicated.[23]

Table 4.15 Fluid Milk Pricing Formula Components

Component	Proportion
1) Price of 16% dairy feed $/tonne (Alberta)	14%
2) Price of Alfalfa hay $/ton (Alberta)	14%
3) Index of Farm Wages (Western Canada)	12%
4) Index of Farm Inputs (Western Canada)	20%
5) Index of Consumer Prices (Canada)	10%
6) Average Weekly Industrial Wage (Alberta)	16%
7) Per Capita Sales of Milk litres/month (Alberta)	14%
Composite	100%

This formula incorporates the major components of market risk into the price of milk. Thus, the dairy producer faces fewer market risks than would producers of commodities that are not priced according to a pricing formula. The major risks faced by the producer are in the area of production

22 Based on the return to 3 year to five year Guaranteed Term Deposits and similar Government of Canada and Alberta bonds.
23 Adapted from Alberta Agriculture. 1986. Economics of Milk Production In Alberta. Page 48.

variation due to health disorders, disease, weather and the natural variability of production. The dairy producer will also face the normal risks associated with the cattle market when he sells cull animals into that market. The market value of replacements can be influenced by the commercial cattle market to the degree that salvage value enters the value of the replacement.[24] As such, the discount rate chosen should reflect a risk-free rate that is free of inflation plus a modest premium for risk of 1/2% to 1%.

At present Government of Canada and Government of Alberta Bonds are paying 9 1/2 % and 9% respectively with inflation estimated to be in the 3% range. Therefore, an appropriate inflation-free rate would be in the range of 6% to 6 1/2%. With an allowance for risk incorporated, the appropriate discount rate would be in the range of 7% to 7 1/2%. A base discount rate of 7 1/2% will be used in this study.

D. Involuntary Disposals

As outlined in Chapter 3, the involuntary replacement of dairy cows can be incorporated into the model by the use of the Markov process. By inserting the probability of involuntary disposal from column two of Table 4.17 into the matrix T from Chapter 3, the following matrix is obtained:

Table 4.16 Markov Matrix

	1	2	3	4	5	6	7	8	9	10
1	0.124	0.135	0.143	0.152	0.170	0.169	0.174	0.177	0.189	1.000
2	0.876									
3		0.865								
4			0.857							
5				0.848						
6					0.830					
7						0.831				
8							0.826			
9								0.823		
10									0.811	0.000

24 For the purposes of this study replacements are valued at the cost of raising the replacement from birth to first calving. The costs used are based on those costs determined by Alberta Agriculture's Rudy Susko in an unpublished study of the Costs of Raising Replacement Dairy Heifers In Alberta.

Table 4.17 Marginal Probability of Replacement[25]

Lactation Number	Invol Replace	Vol. Replace	Total[26] Replace	Prob.[27] Occur
1	12.4	11.0	23.4	76.6
2	13.5	3.6	17.1	82.9
3	14.3	4.1	18.4	81.6
4	15.2	5.3	20.5	79.5
5	17.0	8.0	25.0	75.0
6	16.9	11.9	28.8	71.2
7	17.4	21.6	39.0	61.0
8	17.7	34.9	52.6	47.4
9	18.9	51.1	70.0	30.0
10	20.1	62.6	82.7	17.3
11	21.5	73.5	95.0	5.0
12	24.1	75.9	100.0	0.0

The steady state matrix which denotes the herd composition under various voluntary replacement strategies is obtained from the transition matrix. The adjusted matrix T is obtained first:

Table 4.18 Adjusted Markov Matrix

	1	2	3	4	5	6	7	8	9	10
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.876	-1.000								
3		0.865	-1.000							
4			0.857	-1.000						
5				0.848	-1.000					
6					0.830	-1.000				
7						0.831	-1.000			
8							0.826	-1.000		
9								0.823	-1.000	
10									0.811	-1.000

Solution of the system results in matrix which contains the herd composition "vector" for each replacement strategy.

The matrix c, from which the herd composition, given various voluntary replacement strategies is determined, follows:

25 adapted from Van Arendonk (1985) Livestock Production Science Vol. 13:340

26 Defined as the sum of Involuntary and Voluntary Replacement

27 Defined as 100 minus the total probability of replacement occurring and represents the probability of a particular lactation being completed.

Table 4.19 Herd Composition Matrix

C	1	2	3	4	5	6	7	8	9	10
1	1.000	.5330	.3797	.3046	.2608	.2331	.2141	.2006	.1907	.1834
2		.4670	.3326	.2668	.2285	.2042	.1876	.1757	.1671	.1607
3			.2877	.2308	.1976	.1766	.1622	.1520	.1445	.1390
4				.1978	.1694	.1513	.1390	.1303	.1239	.1191
5					.1436	.1283	.1179	.1105	.1050	.1010
6						.1065	.0979	.0917	.0872	.0838
7							.0813	.0762	.0724	.0697
8								.0629	.0598	.0575
9									.0493	.0474
10										.0384

From the herd composition matrix, the average number of lactations per cow and the average age in years of the cows in the herd given a particular voluntary replacement strategy can be derived. These values follow immediately.

Table 4.20 Predicted Herd Statistics

Lact.	1	2	3	4	5	6	7	8	9	10
ave lact	1.00	1.46	1.90	2.32	2.70	3.05	3.37	3.66	3.93	4.16
ave yrs	3.58	4.16	4.71	5.23	5.71	6.15	6.55	6.91	7.24	7.53
Annual Replace Rate %	100	53.3	37.97	30.45	26.06	23.30	21.39	20.06	19.10	18.87

The vectors Q which follows as Table 4.21, denotes the proportion of cows in each replacement pattern under different replacement strategies. For example, with a voluntary replacement strategy of replacing at the end of lactation three, there are 4.71% of the herd involuntarily replaced at the end of lactation one, 8.98% of the herd involuntarily replaced at the end of lactation two, and 86.31% of the herd replaced at the end of lactation three where the cycle is truncated by choice (i.e. voluntary truncation of the cycle).. This would result in an overall per lactation replacement rate of 37.97%.

The steady state proportion of cows in a given replacement pattern are obtained from the vector Q:

Table 4.21 Proportion Of Cows In A Particular Replacement Pattern

Q	1	2	3	4	5	6	7	8	9	10
1	1.000	.0661	.0471	.0378	.0323	.0289	.0265	.0249	.0237	.0227
2		.9339	.0898	.0720	.0617	.0551	.0506	.0475	.0451	.0434
3			.8631	.0990	.0848	.0758	.0696	.0652	.0620	.0596
4				.7912	.1030	.0920	.0845	.0792	.0753	.0724
5					.7182	.1091	.1002	.0939	.0893	.0859
6						.6391	.0992	.0930	.0884	.0850
7							.5692	.0928	.0882	.0849
8								.5035	.0847	.0815
9									.4433	.0806
10										.3841
Sum	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

V. Results and Analysis

A. The Base Case

Using the vector NPV obtained in Chapter 4, and multiplying by the vector Q from Chapter 4, the following weighted net present values are obtained:

Lact.	1	2	3	4	5	6	7	8	9	10
NPV	11351	14000	14157	13261	12360	11354	10374	9563	8789	8021
% Max	.80	.99	1.00	.94	.87	.80	.73	.68	.62	.56
WNPV	11351	13825	14011	13331	12674	11998	11387	10915	10505	10145
% Max	.81	.91	1.00	.95	.90	.86	.81	.78	.75	.72

Maximization of the vector WNPV results in the determination of the optimal replacement time, which in our case occurs at the end of lactation three where the WNPV reaches a maximum value of $14011.

The ANPV function for the base case is relatively insensitive to the changing values of n (lactation number). The value of the ANPV is 90% or greater of the maximum ANPV value achieved in lactation three (see Figure 5.1) for lactations two, three, four and five.

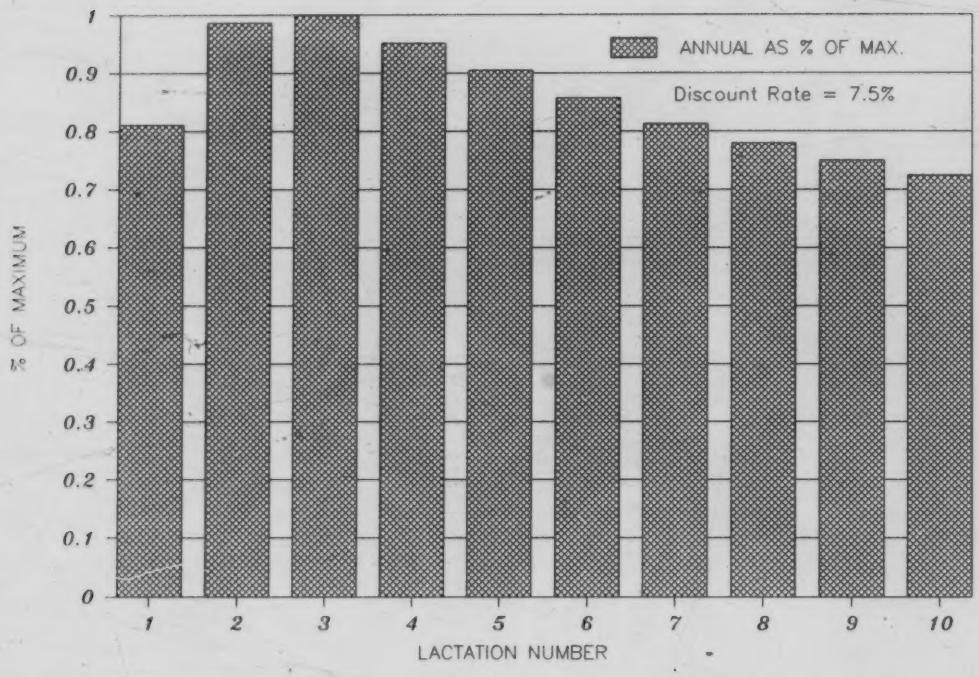

Figure 5.1 Annual ANPV As A Per Cent Of Maximum ANPV For Discount Rate of 7.5%

The model predicts that with voluntary replacement occurring at the end of lactation three that the average age of cows in the herd would be 4.715 years which compares to the actual average age of cows in the provincial dairy herd, as obtained from ADHIS, of 4.78 years.

The model predicts replacement at the end of the third lactation. Data obtained from ADHIS indicates that the average age of disposal of cows from the dairy population in Alberta is 5.3 years of age which is during the cows third lactation. Given the age of disposal, the point of disposal in terms of lactations would be 2.64 lactations.

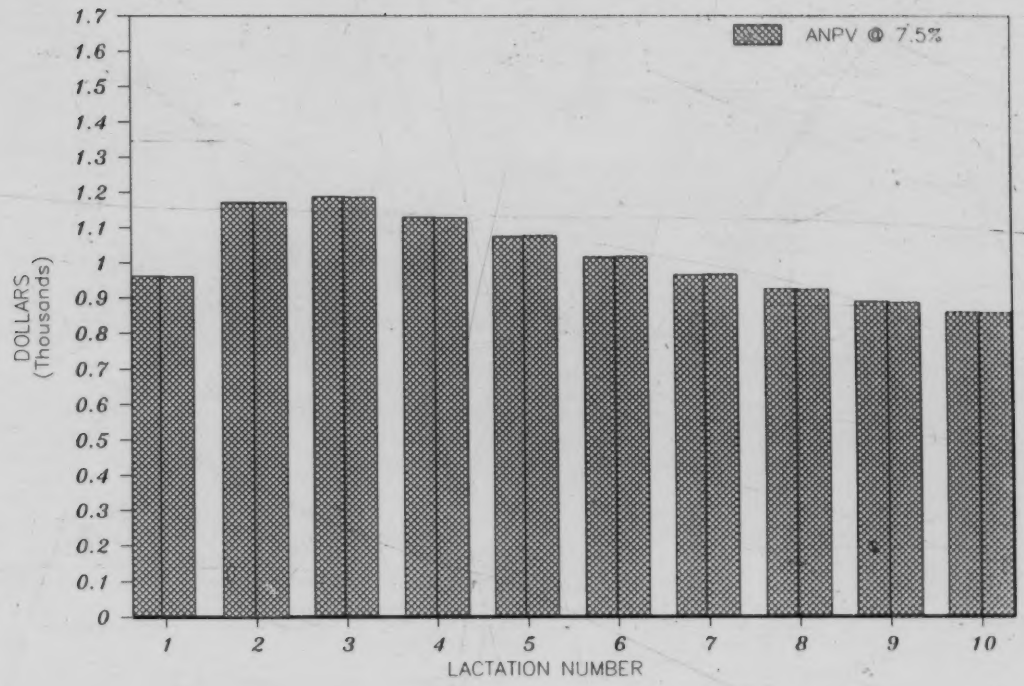

Figure 5.1a Annualized NPV Over Time.

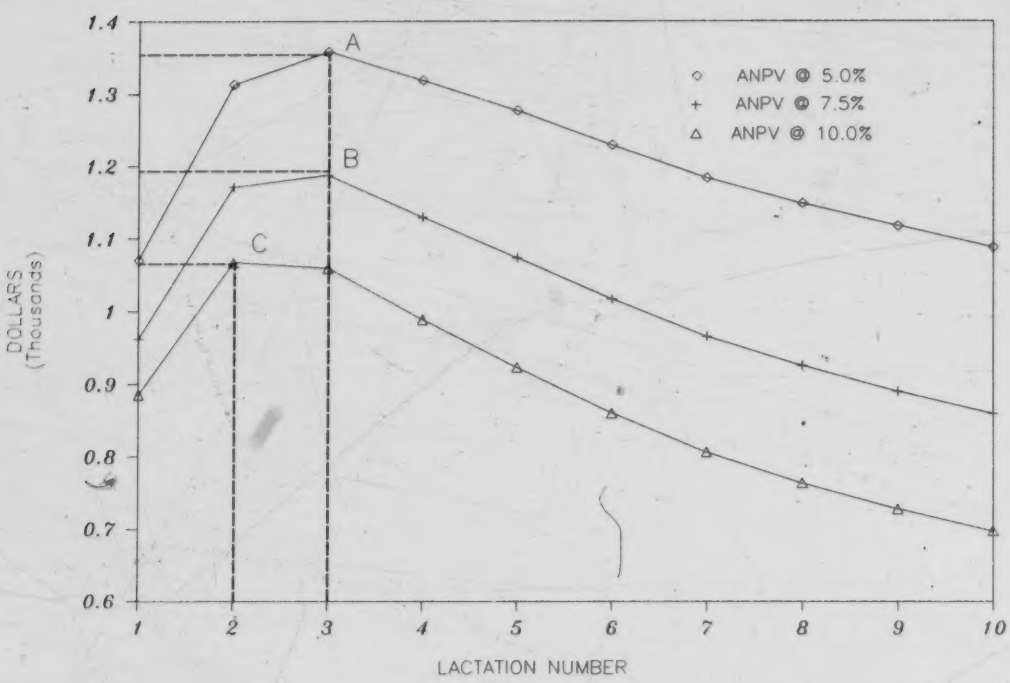

Figure 5.2. Changes in Replacement Time.

B. Discount Rates and Rates of Genetic Improvement:

An increase in the discount rate results in a decrease in the value of the ANPV function. As can be seen in Figure 5.2, an decrease in the ANPV function moves the function downwards.

While the ANPV function decreases due to the higher discount rate, it also reaches its maximum value sooner, resulting in the replacement point occuring earlier (point C in Figure 5.2) than at the lower discount rates.

When the ANPV function increases as the result of a lower discount rate, it moves upwards and shifts the point of maximization to the right (point A in Figure 5.2), resulting in an increase in the replacement time.

As can be seen in Figures 5.3 and 5.4, as the discount rate changes, so does the nature of the ANPV function. Figure 5.4 depicts a discount rate of 5.0%. Here the ANPV is at 90% or greater of the maximum ANPV from lactation two thru lactation six. This is a wider range of lactations than for the base case discount rate of 7.5%. When the discount rate increases to 15.0% (Figure 5.3) the ANPV is at 90% or greater of the maximum ANPV only for lactations two thru four. Thus, as the discount rate increases the ANPV function maximizes earlier but deteriorates quicker than at lesser discount rates.

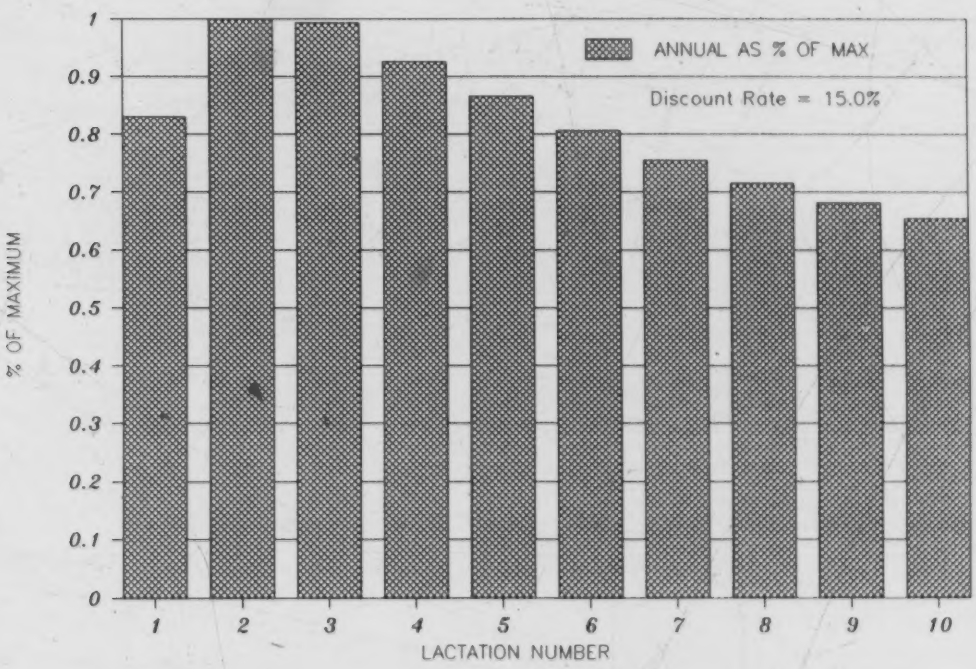

Figure 5.3 Annual ANPV As A Per Cent Of Maximum ANPV For Discount Rate of 15.0%

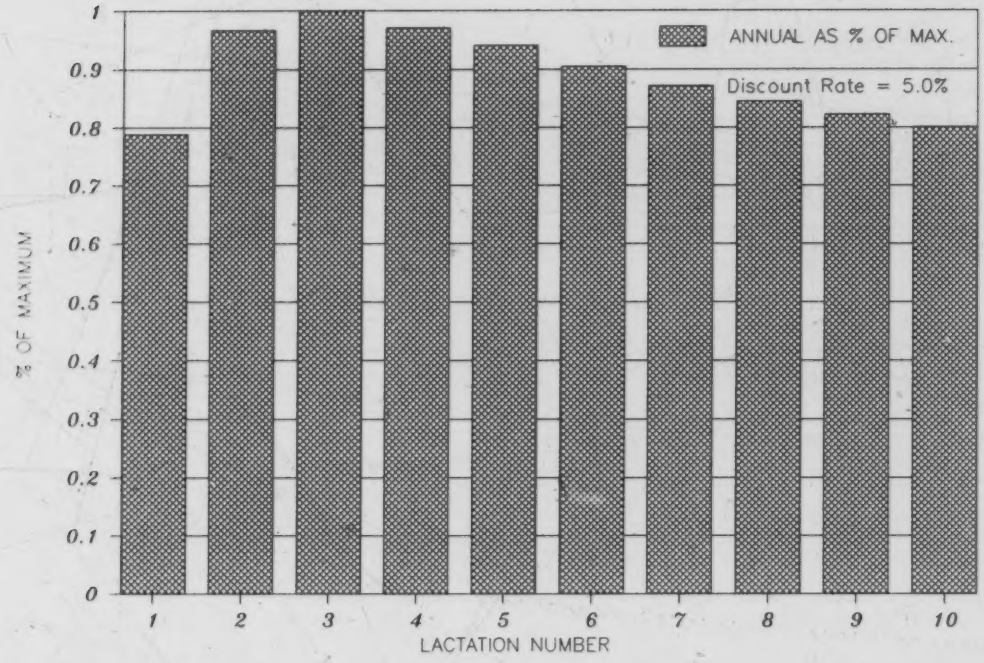

Figure 5.4 Annual ANPV As A Per Cent Of Maximum ANPV For Discount Rate of 5.0%

Table 5.1 Discount Rate and Rate of Genetic Improvement			
Discount Rate	Genetic Improve.	Replacement Time	ANPV
5.0%	0.0%	3	$1219
	0.5%	**3**	**$1359**
	1.0%	3	$1533
	1.5%	3	$1758
	2.0%	3	$2057
7.5%	0.0%	3	$1107
	0.5%	**3**	**$1188**
	1.0%	3	$1281
	1.5%	2	$1391
	2.0%	2	$1530
10.0%	0.0%	2	$1007
	0.5%	**2**	**$1067**
	1.0%	2	$1134
	1.5%	2	$1210
	2.0%	2	$1294
15.0%	0.0%	2	$869
	0.5%	**2**	**$904**
	1.0%	2	$941
	1.5%	2	$982
	2.0%	2	$1025

1. Sensitivity To Discount Rates.

As indicated in Chapter III. Section C. "Determining The Cash Flows", the dairy cow replacement question is comprised of two parts. First is the capital component embodied in the initial cost and terminal value of the cow and value of calf produced and secondly, the annual operating component comprised of milk production

The discount rate affects both parts of this equation. As the discount rate increases, the present value of the stream of operating receipts decreases. But, if the replacement question is viewed as a question of capital intensity, then the higher the discount rate becomes, the greater the value of the capital (the cost of the cow). Thus, as the discount rate increases, the operator would rationally postpone the replacement process by keeping the existing cow longer than he would given a lower discount rate. This characteristic is then offset by the increased value (due to the higher discount rate) of the annuity value of the operating receipts taken in perpetuity. This perpetuity is also growing in value at each stage because of the impact of genetic improvement on the annuities.

The model is sensitive to the discount rate. At a discount rate of 5%, the optimal point of replacement is at the end of lactation three with a ANPV of $1359.

A rise in the discount rate to 7.5% (the base case) leaves the point of replacement unchanged and a decreases the ANPV to $1188.

Increasing the discount rate to 10% decreases the point of replacement to the end of the second lactation and decreases the ANPV to $1067.

Changing the discount rate to 15%, leaves the economical replacement point to the end of lactation two and further decreases the ANPV of the residual net income to a low of $904.

Thus, a change of 10% in the discount rate decreases the ANPV from $1359 to $904 or $455 in total while increasing the point of replacement to the end of the second lactation from the end of the third lactation.

2. Sensitivity To Genetic Improvement Rates.

When no genetic improvement is present the economical replacement point is at the end of lactation three with a ANPV of $1107. By increasing genetic improvement to a rate of 2% per year, the economical replacement point changes to the end of lactation two with a ANPV of $1530.

Again, a large change in the range of genetic improvement alters the ANPV of the residual net cashflow by $423 and shortens replacement cycle by one lactation.

If the discount rate used was very low (i.e. 2.5% or 3%) and the rate of genetic improvement per year was at the high level (2.0%), the replacement cycles would become shorter. The model would explode when the discount rate equaled the rate of genetic improvement. This would result in a 0 value in the denominator of the calculation of the perpetuity value. Effectively, the inclusion of a factor for genetic improvement lowers the discount rate used.

C. Using Varying Levels of Feed and Milk Prices

Table 5.2. Milk Price versus Feed Prices				
Milk Price	Hay	Grain	Replacement Time	ANPV
$39.11	$0.08	$0.20	3	$ 946
	$0.11	**$0.25**	**3**	**$ 701**
	$0.17	$0.30	3	$ 311
$49.11	$0.08	$0.20	3	$1433
	$0.11	**$0.25**	**3**	**$1188**
	$0.17	$0.30	3	$ 798
$59.11	$0.08	$0.20	2	$1926
	$0.11	**$0.25**	**2**	**$1675**
	$0.17	$0.30	3	$1285

1. Sensitivity To Input and Output Prices.

Changes in input and output prices produce replacement timing changes. When milk prices were increased, the replacement timing decreased to the end of the second lactation from the base scenario replacement point of at the end of the third lactation.

Increasing or decreasing feed prices resulted in no change in the point of replacement but rather resulted only in increases or decreases in the ANPV.

D. Varying Replacement Costs and Salvage Values of Cows

Table 5.3. Cost of Replacement and Salvage Values of Cows			
Replacement Cost	Salvage Value Of Cow	Replacement Time	ANPV
$900	Low	3	$1217
	Base	2	$1275
	High	2	$1336
$1000	Low	3	$1182
	Base	2	$1223
	High	2	$1284
$1100	Low	3	$1146
	Base	**3**	**$1188**
	High	2	$1233
$1200	Low	3	$1111
	Base	3	$1152
	High	3	$1193
$1350	Low	3	$1058
	Base	3	$1099
	High	3	$1140

1. Sensitivity To Cost of Replacement

Decreasing the cost of replacements increases the value of the ANPV of the residual cashflows and moves the point of replacement at the end of lactation two. Increasing the cost of the replacements does not affect the replacement cycle while decreasing the ANPV of the residual cashflows.

The range of the ANPV of the residual cashflow is from a high of $1275 to a low of $1099 for a range of $176 with the higher value being associated with the lower priced replacements.

2. Sensitivity To The Salvage Value of the Cow

Changing the salvage value of the base cow results in a range of ANPV of the residual cashflow of $87 with the higher salvage value cows having the low value and the lower salvage value cows the highest value.

The higher salvage value cows produced a range of values of the ANPV of $1336 to $1140 as the replacement cost rose from $900 to $1350. The higher cost heifers were kept one lactation longer than the lower cost heifers.

The low salvage value cows produced a range of ANPV of the residual cashflow of $1217 to $1058 as replacement cost rose from $900 to $1350. The higher cost heifers were kept the same number of lactations as the lower cost heifers.

The model is more sensitive to changes in replacement cost and body weight combined than to the variables independently.

E. Varying Production Levels and Calving Intervals

Table 5.4. Varying Levels of Milk Production and Calving Intervals

Milk Production	Calving Interval	Replacement Time	ANPV
Low 6000 kg./lact.	12.0 months	3	$ 777
	13.5 months	3	$ 748
	15.0 months	3	$ 720
Base 7401 kg./lact.	12.0 months	3	$1228
	13.5 months	3	**$1188**
	15.0 months	3	$1148
High 12,000 kg./lact.	12.0 months	3	$1862
	13.5 months	3	$1805
	15.0 months	2	$1760

1. Sensitivity To Levels of Milk Production.

Increases in milk production increase the value of the ANPV function. While high production increases the ANPV; it does not change the economical replacement point in time.

Decreases in milk production decreases the value of the ANPV but leaves the replacement cycle unchanged at the end of the third lactation.

2. Sensitivity To Changes In Calving Intervals.

The model is sensitive to increasing the calving interval (increasing the length of each lactation) in terms of decreasing the ANPV of residual cashflow values but the economical replacement time is unchanged at all the levels of milk production examined except at the high level of 12,000 kilograms.

F. Replacement Strategies Summarized

Table 5.5 Replacement Strategies Summarized

Case	LACTATION REPLACE	ANP VALUES
Base Case	**3**	**$1188**
12.0 month calving interval	3	$1228
15.0 month calving interval	3	$1148
High Prod. - 12,000 kg.	3	$1805
Low Prod. - 6,000 kg.	3	$ 748
5% Discount Rate	3	$1359
10% Discount Rate	2	$1067
15% Discount Rate	2	$ 904
0% Genetic Improve.	3	$1107
1.0% Genetic Improve.	3	$1281
1.5% Genetic Improve.	2	$1391
2.0% Genetic Improve.	2	$1530
$900 Replacement	2	$1275
$1000 Replacement	2	$1223
$1200 Replacement	3	$1152
$1350 Replacement	3	$1099
Milk Price = $39.11	3	$ 701
Milk Price = $59.11	2	$1675
Feed Price Increase	3	$ 798
Feed Price Decrease	3	$1433
High Salvage Value of Cows	2	$1233
Low Salvage Value of Cows	3	$1146
$900 - High Salvage Value	2	$1336
$1000 - High Salvage Value	2	$1284
$1200 - High Salvage Value	3	$1193
$1350 - High Salvage Value	3	$1140
$900 - Low Salvage Value	3	$1217
$1000 - Low Salvage Value	3	$1182
$1200 - Low Salvage Value	3	$1111
$1350 - Low Salvage Value	3	$1140

1. Summary.

The results are not as sensitive to changes in the individual values of the variables included in the model as they are to variations in combinations of variables. In many of the cases examined, the economical replacement time did not change but the ANP value varied as the variables were allowed to take on a range of feasible values. Overall, the model is insensitive to changes in the variables as the point of economical replacement varied between being at the end of lactation three or the end of lactation two.

For the most part, there appear to be few penalties attached to not following the calculated economic replacement strategy.

In general, it appears that as long as the cow is healthy, calving regularly, and producing at an average level of production, there exists no real need to cull and replace the particular cow. As is the case with many producers, the cows can be allowed to cull themselves by reproductive failure, low production, sickness, injury or death.

VI. Conclusions and Summary

A. Summary

The Alberta Dairy Seminar Committee has identified the economic life of a dairy cow as an important area of concern to the dairy industry of Alberta.

Production data were collected from four co-operating dairy producers. A production prediction equation was developed from this data.

A mathematical algorithm was developed to determine the net present value of the net annual residual cashflows from the dairy cow (including changes in salvage value between lactations and a value for the calf produced). The resultant net present values for each feasible lactation were then evaluated using a finite Markov process to determine the maximum weighted net present value of the production process. This also facilitated the determination of the average age of a herd (in steady state) given a specific maximizing lactation. The proportions of the herd that would be in the feasible lactations was also determined in this process.

Using the milk yields determined and the optimizing algorithm, a model was developed to predict net cashflows from the dairy production process and the point at which the maximum future value occurred. By the decision rule developed, replacement would occur at this point.

Various situations were developed and evaluated in order to allow comparing of the simulation results.

The conclusion reached was that there was no difference between the calculated economic replacement strategy and the current replacement practices being practiced by Alberta dairymen.

The point at which replacement occurs (i.e. at the conclusion of the 3rd., 4th., 5th., or 6th.) lactation does not result in major economic losses to the dairy operation. As previously stated, as long as the dairy cow is productive, healthy and continuing to calve regularly, it is reasonable to keep her in the herd.

B. Conclusions

The objective of the study was to identify the economic replacement strategy for Alberta dairy cows and to subsequently determine if a difference existed between the calculated economic replacement strategy and the current Alberta situation.

The study successfully identified the economic replacement strategy as being one of replacement at the end of lactation three with a new genetically superior cow.

In comparing the calculated economic replacement strategy with the current Alberta situation as evidenced by data obtained from Alberta Dairy Herd Services, it was found that, on average, the dairy producers of Alberta are behaving as would be expected given the assumptions of rational economic behavior incorporated into the model. There are no major differences between point of replacement predicted and that which is occurring in the industry. This is further born out by the determination of the average age of the herd given the particular replacement strategy. The average age of the herd calculated by the model is almost exactly that which is observed in the Alberta industry.

C. Further Research

The model developed and employed in this study made restrictive assumptions in that culling due to knowledge gained about subsequent lactations at the completion of the current lactation has not been incorporated into the model. The model may also be restrictive in that the veterinary costs and rates of change of these costs as well as the rates of involuntary culling were both obtained from the literature as no relevant or reliable data were available to the author on these components.

Further investigation into veterinary costs relative to age and lactation number as well as production level is recommended. The rate of change in veterinary costs as the cow ages can be of importance in the replacement decision process.

The study could be converted into a continuous time study such that within lactation replacement strategies could be determined and evaluated.

While herd demography should be studied more deeply, there appears to be a lack of adequate data to facilitate such studies. Perhaps research effort needs to be expended to develop a more adequate data base.

The establishment of an appropriate lifetime lactation curve needs further study as this curve is an important component of the replacement question. The data available are confounded because both voluntary and involuntary culling are present. There is further confounding because the dairy farmers are making economic decisions within the data set. Experimental data are suggested to resolve this issue.

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VIII. APPENDIX

A. Appendix I. OLS Results - All Cows.

OLS RESULTS

Data set: a:dairy.dat
Dependent variable: PROD

Observations: 2640 Degrees of freedom: 2624
R-squared : 0.519 Rbar-squared : 0.516
Residual SS :4.373E+009 Std. error of est. : 1290.940
Total SS :9.085E+009 F(16 ,2624)=188.4971 P-value=0.00

Var	Coef.	Std. Coef.	Std. Error	t Stat
CONST	1703.4361	0.0000	321.8398	5.2928
AGE	-35.2192	-0.0386	47.0279	-0.7489
INT	-100.6928	-0.0966	17.8130	-5.6527
MILK	24.6382	0.7370	0.5790	42.5465
BFAT	-540.9538	-0.1181	63.1496	-8.5662
D2	898.0042	0.2036	85.4321	10.5113
D3	1461.6843	0.2955	124.8124	11.7110
D4	1498.1974	0.2559	170.3822	8.7931
D5	1430.8752	0.2012	218.9325	6.5356
D6	1234.0803	0.1402	272.6154	4.5268
D7	1253.8706	0.1063	335.1883	3.7407
D8	1593.2653	0.1036	402.1491	3.9618
D9	1495.3037	0.0698	485.6216	3.0791
D10	1268.5747	0.0398	615.2029	2.0620
D11	2218.8230	0.0402	895.2204	2.4785
D12	1232.3630	0.0129	1399.5943	0.8805

B. Appendix II. OLS Results - 4 + Lactations.

OLS RESULTS

Data set : a:cows4.dat
Dependent variable: PROD

Observations: 1620 Degrees of freedom: 1604
R-squared : 0.496 Rbar-squared : 0.491
Residual SS : 2.502E+009 Std. error of est. : 1248.944
Total SS : 4.960E+009 F(16 ,1604)=105.0461 P-value=0.00

Var	Coef.	Std. Coef.	Std. Error	t Stat
CONST	1104.0647	0.0000	406.5746	2.7155
AGE	-88.8451	-0.1110	60.2356	-1.4749
INT	-83.3103	-0.0810	23.5897	-3.5316
MILK	24.0205	0.6950	0.8047	29.8501
BFAT	-417.8504	-0.0932	81.2215	-5.1445
D2	968.9205	0.2093	122.4543	7.9125
D3	1669.3501	0.3642	163.9484	10.1821
D4	1948.3320	0.4308	216.9615	8.9800
D5	1931.9964	0.3584	276.4489	6.9886
D6	1797.3016	0.2720	341.7042	5.2598
D7	1887.7776	0.2148	414.5106	4.5542
D8	2277.0302	0.1994	490.6084	4.6412
D9	2214.2481	0.1397	577.8765	3.8316
D10	2036.3653	0.0865	704.7815	2.8893
D11	3000.9394	0.0737	964.0102	3.1129
D12	2118.4518	0.0300	1430.8936	1.4805

C. Appendix III. OLS Results - 5 + Lactations.

OLS RESULTS

Data set : a:cows5.dat
Dependent variable: PROD

Observations: 1219
R-squared: 0.489
Residual SS: 1.910E+009
Total SS: 3.735E+009

Degrees of freedom: 1203
Rbar-squared: 0.482
Std. error of est.: 1260.055
F(16 ,1203)=76.6263 P-value=0.00

Var	Coef.	Std. Coef.	Std. Error	t Stat
CONST	824.4844	0.0000	498.4302	1.6541
AGE	-20.5963	-0.0272	73.1703	-0.2814
INT	-87.2005	-0.0813	27.8222	-3.1342
MILK	24.1789	0.6658	0.9514	25.4121
BFAT	-415.8598	-0.0873	101.3691	-4.1024
D2	835.8940	0.1732	149.9851	5.5731
D3	1405.3713	0.2887	200.5257	7.0084
D4	1767.2721	0.3670	264.4506	6.6828
D5	1757.0593	0.3664	333.9928	5.2607
D6	1541.4569	0.2652	411.9913	3.7414
D7	1560.2684	0.2031	497.3623	3.1370
D8	1877.6796	0.1887	584.6997	3.2113
D9	1745.5008	0.1266	678.9043	2.5710
D10	1497.8618	0.0732	811.5690	1.8456
D11	2383.9375	0.0674	1065.9481	2.2364
D12	1431.9580	0.0234	1520.7792	0.9415

D. Appendix IV. Frequency Distribution of Lactations.

Lact	N	Mean	St Dev	Variance	Minimum	Maximum
1	833	6214.1	1870.5	.34989E+07	475	14503
2	609	6729.7	1804.1	.32549E+07	583	12642
3	451	7188.4	1942.9	.37748E+07	388	13199
4	299	7289.7	1809.2	.32733E+07	1770	12925
5	197	7094.5	1842.9	.33962E+07	195	12124
6	125	6977.2	1688.7	.28518E+07	2510	13748
7	68	6992.3	2092.9	.43801E+07	2462	13950
8	40	7056.5	2183.0	.47654E+07	3000	11023
9	20	7406.7	1431.9	.20503E+07	4186	10355
10	10	6611.1	2460.9	.60561E+07	1064	10015
11	3	6372.7	2694.7	.72612E+07	3272	8148
12	1					

1. Appendix IVa. Distribution Of Commercial Disposals.

Commercial Herd Disposals		
Lactation Number	Number Of Disposals	Cummulative Disposals in %
1	224	26.89%
2	158	45.86%
3	152	64.11%
4	102	76.35%
5	72	84.99%
6	57	91.84%
7	28	95.20%
8	20	97.60%
9	10	98.80%
10	7	99.64%
11	2	99.88%
12	1	100.00%

2. Appendix IVb. Distribution Of ADHIS Disposals

ADHIS Herd Disposals		
Lactation Number	Number Of Disposals	Cummulative Disposals as %
1	12329	26.34%
2	12238	52.50%
3	8456	70.56%
4	5776	82.90%
5	3760	90.95%
6	2153	95.55%
7	1157	98.02%
8	536	99.17%
9	240	99.68%
10+	150	100.00%

E. Appendix V. Dairy Ration to Production Relation

Grain Feeding Guide With Good Forage[28]						
		Concentrate per Day				
		Butterfat				
Milk per Day		3.5%		4.0%		5.0%
lb.	kg.	lb.	kg.	lb.	kg.	lb.
20	9.1	3	1.4	5	2.3	7
25	11.3	6	2.7	7	3.2	10
30	13.6	8	3.6	9	4.1	12
35	15.9	11	5	12	5.4	15
40	18.1	13	5.9	14	6.4	18
45	20.4	16	7.3	18	8.2	22
50	22.7	19	8.6	21	9.5	26
55	25.0	23	10.4	25	11.3	30
60	27.2	27	12.2	29	13.2	34
65	29.5	31	14.1	33	15.0	
70	31.7	35	15.9	37	16.8	
75	34.0	39	17.7			
80	36.3					
85+	38.6+					

F. Appendix VI. Grain Feeding Guide With Good Forage

Grain Feeding Guide With Good Forage Used In The Model	
Milk per Day	Concentrate per Day
kg.	kg.
5	0
9	2
11	3
14	4
16	5
18	6
20	8
23	10
25	11
27	13
30	15
32	17
34	19

28 adapted from Alberta Agriculture Farm Mamagement Guide. 1976. page 200.